ISSN 1063-7826, Semiconductors, 2012, Vol. 46, No. 8, pp. 1051–1058. © Pleiades Publishing, Ltd., 2012.
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Photoelectric Determination of the Series Resistance of Multijunction Solar Cells

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Submitted February 13, 2012; accepted for publication, February 17, 2012

Abstract—A method for determining the series resistance R_s of multijunction solar cells is suggested and substantiated. The method uses the presence of a maximum in the dependence of the efficiency on the sunlight concentration ratio $\eta(X)$ or in that of the operating voltage on the photogenerated current, $V_m(J_g)$. The study employs the concept that, in a limited but practically important range of photogenerated currents (up to the maximum η), the series resistance can be represented by a fixed quantity that is linear and independent of J_g. It is analytically substantiated that this resistance can be found from the formula $R_s = (E/J_g)_{\eta = \max}$, where $E = AkT/q$ and A and J_g are local values of the ideality factor and photogenerated current at the maximum η (or V_m). It is shown that the value of R_s, determined by this method, is independent of the spectral composition of the incident light, which was experimentally confirmed in a study of the photovoltaic characteristics of triple-junction InGaP/GaAs/Ge solar cells. The method is suitable for both multi- and single-junction photoelectric converters.

DOI: 10.1134/S1063782612080143

1. INTRODUCTION

One of the important factors affecting the photoelectric conversion efficiency (η) of concentrator multijunction (MJ) solar cells (SCs) of various types are the internal resistive losses covered by the term "series resistance." Because of the presence of this resistance, the increase in the photoconversion efficiency with increasing illumination intensity (sunlight concentration ratio X) gives way to a decaying portion, with a maximum formed [1–3].The maximum value is also reached by the operating voltage (voltage at the optimal-load point) V_m (Fig. 1) [4].

The contribution to the resistive loss is made both by the transverse resistance of the SC layers and by the longitudinal (lateral) spreading resistance in the upper layer [5–7]. There exist, at least, two main approaches toward analyzing series resistance: (i) simulation of a multi-unit equivalent circuit [5] or a combination of units (cells) [8–10] and (ii) representation of the photoelectric converter (PEC) as an electrical circuit constituted by series-connected elements, i.e., a generating part (constituted by photovoltaic p–n junctions) and a resistive part (lumped equivalent of the resistive loss) [11]. When the contribution of the spreading resistance is dominant, the lumped equivalent is a nonlinear resistance dependent on the illumination intensity. In [5], an approach towards analysis of a multi-unit model was developed. The model can describe this behavior and determine the basic properties of the distributed series resistance.

In this study, we use the concept that, in practically important cases (up to the maximum efficiency), this nonlinear lumped equivalent can be replaced with a fixed (linear and independent of the illumination intensity) series resistance R_s [12]. We consider a method for determining this series resistance in MJ SCs, both balanced (with equal currents photogener-

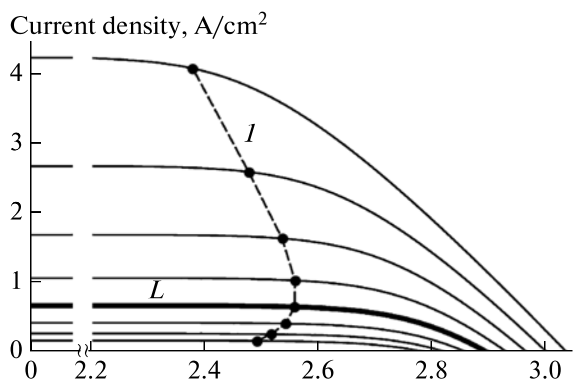

Fig. 1. Current–voltage characteristics of an MJ SC at various light concentration ratios. (*1*) A line connecting the maximum-power points. L is the characteristic corresponding to the maximum value of the operating voltage V_m.

ated by the photovoltaic p–n junctions) and unbalanced in photogenerated current (with unequal photogenerated currents).

Based on the existence of a maximum in the operating voltage, we analytically derived and extended to the efficiency maximum the base formula for determining the series resistance. We also substantiate our analytical results by experimental data for MJ SCs based on GaInP/Ga(In)As/Ge structures. It is known [1, 2, 13] that sunlight converters with the highest efficiency at present have been developed on the basis of these structures.

2. CURRENT–VOLTAGE CHARACTERISTIC OF A MULTIJUNCTION SOLAR CELL: GENERAL, REDUCED, AND IDEALIZED FORM

Both the current–voltage characteristic $V(J)$ [$(I$–$V)$] and any photovoltaic characteristic of an MJ SC can be approximated with a set of segments [14]. In each segment, the $V(J)$ [I–V] characteristic of the generating part is formed by summation of the voltages at the photovoltaic p–n junctions. These voltages depend on the dominant charge transport mechanism (of diffusion or recombination type) in the considered segment. As a result, the $V(J)$ characteristic of a segment (with consideration for the resistive loss simulated by a series-connected linear resistance) has the form

$$V = \frac{kT}{q}\sum_{i=1}^{n}\ln\left[\frac{J_{gi}-J}{J_{si}}\right]^{A_i} - JR_s = \frac{kT}{q}\ln\left(\prod_{i=1}^{n}\left[\frac{J_{gi}-J}{J_{si}}\right]^{A_i}\right) - JR_s, \quad (1)$$

where i is the sub-element number; J_{si} are the pre-exponential factors (“saturation” currents); A_i, are the diode (ideality) factors equal to unity (dominant diffusion mechanism) or two (recombination mechanism); J_{gi} are the photogenerated currents; n is the number of sub-elements; R_s is the series resistance; q is the elementary charge; T is the temperature; and k is the Boltzmann constant.

If the lowest of the photogenerated currents is denoted by J_g, i.e., $J_g = \min\{J_{g1}, J_{g2}, ..., J_{gn}\}$, then (1) can be represented as the sum of three terms:

$$V = E\ln\left(\frac{J_g - J}{J_s}\right) - JR_s + V_a. \quad (2)$$

In the first term, $E = AkT/q$ is the volt diode factor; $A = \sum_{i=1}^{n} A_i$, and $J_s = \sqrt[A]{\prod_{i=1}^{n} J_{si}^{A_i}}$. The second term is resistive. The third accounts for imbalance of the photogenerated currents:

$$V_a = \frac{kT}{q}\ln\left(\prod_{i=1}^{n}\left[\frac{\kappa_i J_g - J}{J_g - J}\right]^{A_i}\right), \quad (3)$$

where $\kappa_i = J_{gi}/J_g \geq 1$ are the current imbalance coefficients.

It is noteworthy that, in practice, the lowest of the photogenerated currents,

$J_g = \min\{J_{g1}, J_{g2}, ..., J_{gn}\}$, is equal to the short-circuit current J_{sc} because the condition $J_{sc} < V_{oc}/R_s$ is satisfied (V_{oc} is the open-circuit voltage).

At total balance (all $\kappa_i = 1$), the voltage V_a becomes zero and the $V(J)$ characteristic (2) takes the same form as that for a single-junction SC [5]. With growing imbalance (increasing κ), V_a becomes higher. Thus, the first term in formula (2) is reduced to the balanced (single-junction) form, the second is the ordinary voltage on the series resistance, and the third (V_a) characterizes the imbalance of the photogenerated currents.

As already noted, the series resistance is the reason for the formation of the maximum in the $\eta(X)$ characteristic. For the same reason, the operating voltage V_m also has a maximum (Fig. 1). Thus, these maxima are related. However, it is more convenient to analytically describe the maximum by using the following dependences of the operating voltage: $V_m(J_g)$, $V_m(J_m)$, and $V_m(J_g - J_m)$. The algebraic and differential relations necessary for further presentation of the essence of the method are given in Appendix. To derive these relations, we use the following idealization: the total balance of photogenerated currents ($V_a = 0$) and zero-resistance case ($R_s = 0$). Then (2) takes the form

$$V = E\ln\left(\frac{J_g - J}{J_s}\right). \quad (4)$$

3. BALANCED MULTIJUNCTION SOLAR CELL

3.1. Approximate Account of the Series Resistance

It is assumed that, when the effect of the series resistance on the operating point (V_m, J_m) is taken into account in the balanced case ($V_a = 0$ in Eq. (2)), the following condition is satisfied (Fig. 2):

$$J_m \approx J_{m0}, \quad (5)$$

(here and hereinafter the subscript “0" designates the zero-resistance case).

Then, for the operating point, formula (2) takes the form

$$V_m = V_{m0} - J_m R_s, \quad (6)$$

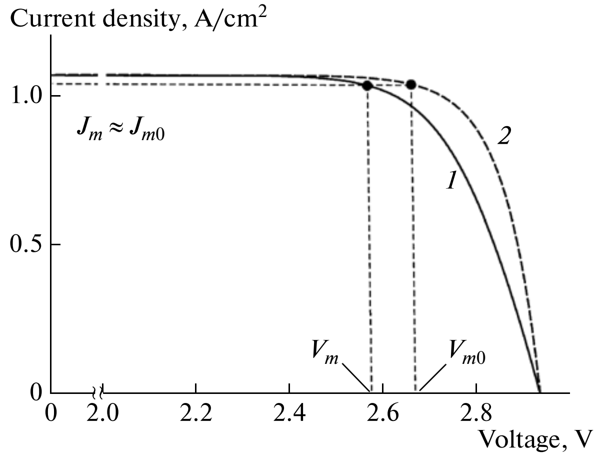

Fig. 2. Light (load) characteristic $V(J)$ and the position of the operating point, found with R_s (*1*) taken into account and (*2*) disregarded.

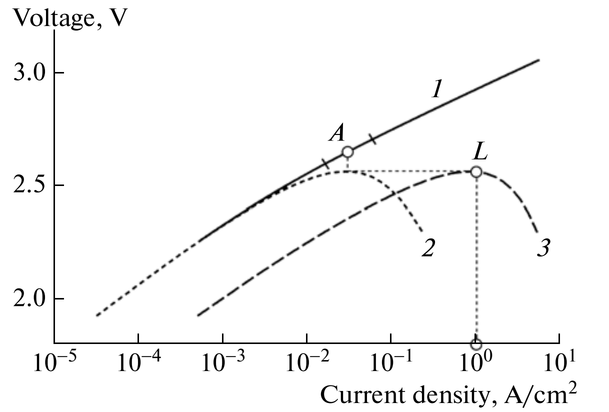

Fig. 3. Mutual positions of photovoltaic characteristics in a current-balanced MJ SC: (*1*) $V_{oc}(J_g)$, (*2*) $V_m(J_g - J_m)$, and (*3*) $V_m(J_g)$. The characteristics $V_{oc}(J_g)$ and $V_m(J_g - J_m)$ coincide in the zero-resistance portion.

where the current dependence of the zero-resistance voltage V_{m0} is given by formulas (A.3) and (A.5), depending on the choice of the current scale (J_m, $(J_g - J_m)$ or $J_g(\propto X)$) and the current dependence of the resistive summand J_mR_s on different scales is obtained from (A.4) and (5).

Thus, from (6) follows the current dependence of the operating voltage V_m on different current scales: J_m, $(J_g - J_m)$ or $J_g(\propto X)$.

When the first summand V_{m0} in (6) increases (see (A.3) and (A.5)), the second decreases, with a resistance-related maximum formed. The position of the maximum in V_m on different current scales is found by equating the corresponding derivatives to zero. These derivatives are related to each other via dJ_g/dJ_m and $d(J_g - J_m)/dJ_m$, which are nonzero as indicated by (A.8a), (A.8b), and (A.8c). Therefore, the conditions $dV_m/dJ_g = 0$ and $dV_m/d(J_g - J_m) = 0$ are equivalent to the condition $dV_m/dJ_m = 0$, which is simpler and more convenient for calculations.

Differentiation of (6) with consideration for (5) and (A.7c) yields

$$\frac{dV_m}{dJ_m} = \frac{dV_{m0}}{dJ_{m0}} - R_s = \frac{E}{J_g} - R_s, \qquad (7)$$

whence follows the basic relation underlying the photoelectric method for determining the series resistance (linear lumped equivalent of resistance):

$$R_s = \frac{E_L}{J_{gL}}, \qquad (8)$$

where J_{gL} is the photogenerated current J_g at which the maximum value of the operating voltage V_m is reached, and E_L is the local volt diode factor corresponding to the given voltage.

Thus, the basic principle of R_s determination consists in finding a photogenerated current value J_{gL} at which the maximum in V_m, and the corresponding value of the local volt diode factor E_L, are observed.

3.2. Determination of the Series Resistance

To find the local volt diode factor, we can use two characteristics, $V_{oc}(J_g)$ and $V_m(J_g - J_m)$. As noted in the Appendix, the characteristic $V_{oc}(J_g)$ (A.1) is interpreted as the zero-resistance $V_m(J_g - J_m)$ (A.3). Accordingly, the required value of E_L is equal to the logarithmic slope ratio $\Delta V_{oc}/\Delta \ln(J_g)$ at the point of the maximum in V_m (Fig. 3, point A). Figure 3 illustrates the above-described method for finding the series resistance from expression (8), with the dependences $V_{oc}(J_g)$, $V_m(J_g)$, and $V_m(J_g - J_m)$ used. The quantity J_{gL} is the current at point L, and E_L is found from the slope ratio of the characteristics $V_{oc}(J_g)$ near point A.

It can be seen from the way used to derive the basic formula (8) and from the procedure employed to find the values of J_{gL} and E_L (Fig. 3) that the method suggested for determining the series resistance is entirely valid for single-junction and balanced MJ SCs. The method can also be extended to the case of unbalanced MJ SCs. This generalization is based on taking into account the imbalance correction V_a (3), which was zero in the balanced case.

Thus, determination of the series resistance R_s includes the following stages: finding J_{gL} from $V_m(J_g)$, determining E_L by comparing $V_m(J_g - J_m)$ and $V_{oc}(J_g)$, and applying the basic formula (8).

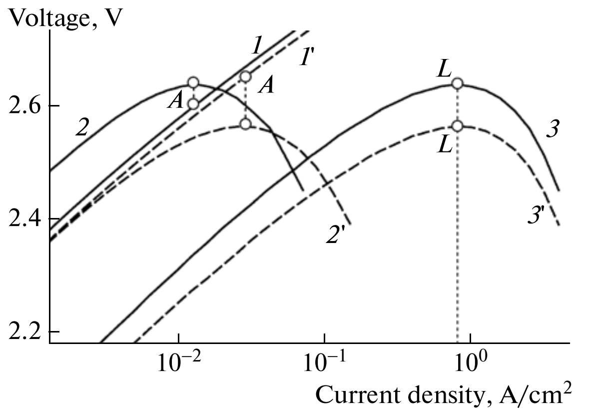

Fig. 4. Mutual positions of photovoltaic characteristics in an unbalanced (solid lines) and a balanced (dashed lines) MJ SC. (*1, 1'*) $V_{oc}(J_g)$, (*2, 2'*) $V_m(J_g - J_m)$, and (*3, 3'*) $V_m(J_g)$. In the unbalanced case, the characteristic $V_m(J_g - J_m)$ is shifted upwards in the zero-resistance portion relative to $V_{oc}(J_g)$, in contrast to the balanced case.

4. UNBALANCED MULTIJUNCTION SOLAR CELL

For a photogenerated current imbalance, in contrast to the balanced case (see Section 3), it is necessary to take into account the imbalance voltage V_a (3). Because the current dependences of V_m and V_{oc} are used to find R_s from formula (8), it is necessary to determine the corresponding imbalance corrections ($V_{a,m}$, $V_{a,oc}$).

In the operating mode ($J = J_m$, $V = V_m$), the imbalance correction to the operating voltage, $V_{a,m}$, can be approximated with a constant. Indeed, the experimentally observed load characteristics have approximately the same (almost rectangular) forms for balance and imbalance. Therefore, it is reasonable to assume that the relationship between J_g and J_m, specified by approximation (5) and formula (A.4), is also valid for the unbalanced case. Then it follows from (3), in which $J = J_m \approx J_{m0}$, that

$$V_{a,m} = (kT/q)\ln\left(\prod_{i=1}^{n}[(\kappa_i - 1)V_{m0}/E + \kappa_i]^{A_i}\right). \quad (9)$$

This variant of formula (3) shows no explicit dependence on J_g, whereas the remaining weak implicit dependence (via V_{m0}) makes it possible to approximate $V_{a,m}$ with a constant (within a limited range of currents).

In the open-circuit mode ($J = 0$, $V = V_{oc}$), it follows from (3) that the imbalance correction to the open-circuit voltage has the form

$$V_{a,oc} = (kT/q)\ln\prod_{i=1}^{n}[\kappa_i]^{A_i}, \quad (10)$$

i.e., it is strictly constant.

It is noteworthy that, as can be seen in (3), the imbalance voltage V_a becomes zero in two cases; e.g., at current balance (all $\kappa_i = 0$) and for the dark characteristic ($J_g = 0$), which provides coincidence of the following three dependences: light $V_{oc}(J_g)$, light zero-resistance $V_m(J_g - J_m)$, and dark zero-resistance $V(J)$ (see Appendix).

In imbalance, there is no coincidence of this kind: the two light characteristics $V_{oc}(J_g)$ and $V_m(J_g - J_m)$ are shifted relative to the dark characteristic $V(J)$ by unequal values of $V_{a,m}$ and $V_{a,oc}$. However, the fact that $V_{a,oc}$ is a constant and $V_{a,m}$ is approximated with a constant in a limited range of currents leaves the derivation of the basic formula (8) and its final form unchanged. Accordingly, the procedure for finding the local volt diode factor (E_L) from the slope ratio of the characteristic $V_{oc}(J_g)$ at point A and the photogenerated current J_{gL} at point L (see Fig. 4) remains unchanged.

Expressions (9) and (10) enable estimation of $V_{a,m}$ and $V_{a,oc}$ for a triple-junction InGaP/GaInAs/Ge SC in whose Ge sub-element, $A_3 = 1$ and $\kappa_3 \approx 1.5$–2 [15], and the currents of the GaInP and GaInAs sub-elements are approximately the same [16] (κ_2, $\kappa_1 \approx 1$), with the charge transport mechanism possibly being either of the diffusion (A_2, $A_1 = 1$) or recombination type (A_2, $A_1 = 2$). According to the estimate, $V_{a,m} \approx 0.07$ V and $V_{a,oc} \approx 0.02$ V and the discrepancy between $V_{oc}(J_g)$ and the zero-resistance $V_m(J_g - J_m)$ is 0.05 V. Consequently, $V_{a,m}$ is ~20% relative to the range of variation of V_m, which is ~0.5 V (Fig. 5). In turn, $V_{a,oc}$ is ~3% relative to the corresponding range for V_{oc} (~0.5 V, Fig. 5). Thus, imbalance corrections of this kind should be taken into account for triple-junction InGaP/GaInAs/Ge SCs.

5. USE OF THE EFFICIENCY CHARACTERISTIC η(X)

To determine R_s, it is possible to use, in addition to V_m, the efficiency characteristic $\eta(X)$, which also has a maximum because of the presence of a resistive loss. For this purpose, it is convenient to introduce an auxiliary quantity, the efficiency voltage

$$V_\eta = \frac{J_m V_m}{J_g}, \quad (11)$$

which is directly proportional to the efficiency $\eta = J_m V_m / P_{inc} = V_\eta / V_{conv}$. In this case, the proportionality coefficient $V_{conv} = P_{inc}/J_g = P_{inc,e}/J_{g,e}$ is independent of the emission intensity ($P_{inc,e}$ is the unit (at the concentration ratio $X = 1$) light power density, and $J_{g,e}$ is the unit ($X = 1$) photogenerated current density.

Here, the efficiency voltage can be expressed in terms of the operating voltage. For example, in the zero-resistance and balanced case, according to (A.4),

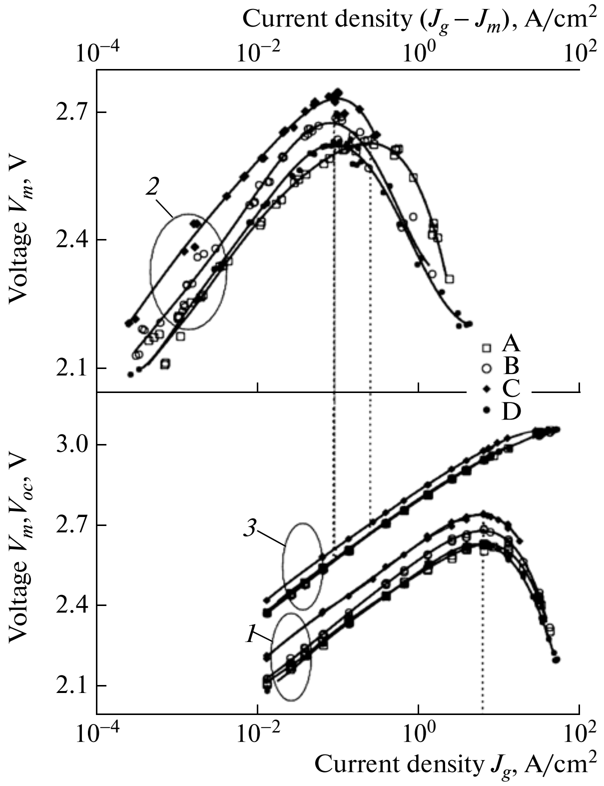

Fig. 5. Experimental photovoltaic characteristics (*1*) $V_m(J_g)$, (*2*) $V_m(J_g - J_m)$, and (*3*) $V_{oc}(J_g)$ for different illumination spectra (A, B, C, D).

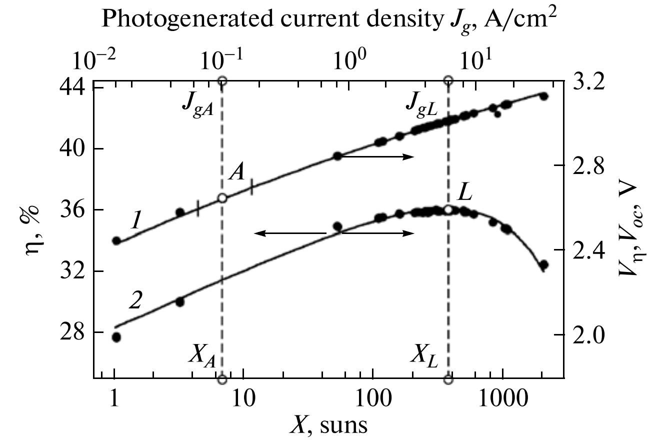

Fig. 6. Determination of the series resistance from a set of two characteristics: (*1*) dependence of $\eta(\propto V_\eta)$ on X (or J_g) and (*2*) dependence of V_{oc} on X (or J_g). Solid lines correspond to the calculation; points correspond to the experiment.

$J_{m0}/J_g = V_{m0}/(V_{m0} + E)$, whence follows from (11) the relation $V_{\eta 0} = V^2_{m0}/(V_{m0} + E)$, simplified to $V_{\eta 0} = V_{m0} - E$ if the voltage diode factor is small as compared with the operating voltage. $E \ll V_{m0}$. The imbalance correction (3) to the efficiency voltage $V_{a,\eta}$ is approximated with a constant (in a limited range) by analogy with $V_{a,m}$.

Because the resultant shift of V_η relative to V_m is approximated with a constant, the essence of the method based on differentiation of the operating voltage (see Section 3.1) remains unchanged. Therefore, all the results following from the presence of a maximum in the operating voltage (see Section 4) can be transferred to the efficiency voltage and, accordingly, to the efficiency. The series resistance can still be found by using the basic formula $R_s = E_L/J_{gL}$, where J_{gL} is the photogenerated current at which the efficiency maximum is reached, and E_L is the corresponding volt diode factor. It is noteworthy that J_{gL} can be found from either $\eta(J_g)$ or $V_m(J_g)$.

The way in which the series resistance is determined according to the basic formula by means of $\eta(X)$ and related characteristics is illustrated by Fig. 6. The value of J_{gL} is found from the concentration ratio X_L (point L has $X_L = 400$ and $\eta = 36.2\%$ in Fig. 6) at which the efficiency is at a maximum, $J_{gL} = X_L J_{g,e}$. The value of E_L is equal to the slope ratio of the characteristic $V_{oc}(X)$ at point A (Fig. 6): $E_L = \Delta V_{oc}/\Delta \ln(X)|_{X = X_A}$, with the position of point A on the concentration scale calculated as $X_A = (J_{gL} - J_{mL})/J_{g,e}$, where J_{mL} is the operating current corresponding to the maximum efficiency. Other scales can also be used: J_g instead of X, and V_η instead of η (Fig. 6).

6. EXPERIMENTAL IMPLEMENTATION OF THE METHOD

It was shown in Section 4 that, in the approximations we used, $J_m \approx J_{m0}$ (5) and V_a = const, the basic formula (8) is derived in the same way for MJ SCs balanced and unbalanced in terms of photogenerated current, with the derivation being independent of the imbalance correction V_a. Consequently, the result obtained when determining the series resistance, must be independent of the spectral composition of the incident light. This circumstance may be of practical use for determining R_s in experimental studies of MJ SCs on simulators of various classes.

A triple-junction InGaP/GaAs/Ge SC fabricated by metalorganic vapor-phase epitaxy (MOVPE) [17] served as the object of study.

We measured several sets of $V(J)$ characteristics at various spectral compositions of incident light and plotted the photovoltaic characteristics $V_m(J_g)$, $V_m(J_g - J_m)$, and $V_{oc}(J_g)$ (Fig. 5).

The measurements were made at room temperature on a simulator with a pulsed xenon lamp and correcting light filters [18]. The practically measured short-circuit current values (even at the maximum lamp intensity) were considered equal to the photoge-

Results of determining the series resistance R_s of the sample under study at different spectral compositions of incident light

Spectrum	J_{gL}, A cm^{-2}	J_{gA}, A cm^{-2}	E_L, V	R_s, 10^{-3} Ω cm^2
A	6.76	0.26	0.092	13.6
B	6.78	0.092	0.097	14.3
C	6.70	0.095	0.092	13.8
D	6.75	0.096	0.096	14.2

nerated currents (J_g). The measurements were conducted with four types of spectra: A, B, C, and D. Spectrum A, standard AM1.5D; spectrum B, unfiltered spectrum of the pulsed xenon lamp; spectrum C, obtained with a KG-2 red light filter, rich in light of the blue spectral range (300–650 nm); and spectrum D, obtained with an RG-8 blue light filter, rich in light of the red spectral range (650–1000 nm).

As expected, the maxima of the dependences $V_m(J_g)$ lie at approximately the same photogenerated current (see values of J_{gL} in the table and curves *1* in Fig. 5). The positions of the maxima of the $V_m(J_g - J_m)$ characteristics are somewhat different (see J_{gA} in the table and curves *2* in Fig. 5); however, the corresponding values of the slope ratio $\Delta V_{oc}/2.3\Delta\log(J_g) = E_L$ in the $V_{oc}(J_g)$ characteristic are almost the same (E_L in the table, curves *3* in Fig. 5).

The observed positions of the maxima in the characteristics $V_m(J_g)$ agree with the approximations we made. A certain difference in the positions of the maxima in the $V_m(J_g - J_m)$ characteristics has no effect on the results of R_s determination (see the table). It can be seen in the table that the series resistance is $(14.3 \pm 0.3) \times 10^{-3}$ Ω cm^2, i.e., the relative error of the method is 2%.

It follows from the results obtained that the photoelectric technique we developed is applicable to MJ SCs, with the result of determination of the series resistance being independent of the spectral composition of the incident light (both standard (AM0, AM1.5, etc.) and nonstandard spectra can be used).

7. CONCLUSIONS

A new method for determining the series resistance of solar cells, which characterizes the resistive loss in the practically important range of illumination intensities, up to the maximum efficiency or operating voltage, was suggested and substantiated.

An expression for experimental determination of the series resistance was analytically derived: $R_s = E_L/J_{gL}$, where $E_L = AkT/g$ and A and J_g are the local ideality factor and photogenerated current at the illumination intensity at which the efficiency or the working voltage (V_m) are at a maximum. It is noteworthy that, in practice, the photogenerated current is equal to the short-circuit current because the condition $J_{sc} < V_{oc}/R_s$ is commonly satisfied. The value of E is found from the local slope ratio of the $V_{oc}(J_g)$ characteristic at a chosen point, i.e., this is done without using the dark characteristic $V(J)$.

The method can be used with both standard (AM0, AM1.5, etc.) and nonstandard spectral compositions of incident light. The series resistance was determined at different spectral compositions of incident light for a triple-junction GaInP/GaAs/Ge solar cell grown by MOVPE technology. The resulting values of the series resistance were almost the same (with a relative error of ~2%). It was shown that the method can be used to analyze experimental results obtained in studies of the current–voltage characteristics of multijunction solar cells exposed to light of varied spectral composition.

The photoelectric method based on the presence of maxima for η or V_m is also applicable to single-crystal single-junction photoelectric converters and, probably, also to solar cells whose load characteristic can be approximated with an exponential.

APPENDIX

BASIC RELATIONS FOR THE IDEALIZED (ZERO-RESISTANCE AND BALANCED) CASE

1. ALGEBRAIC RELATIONS

Idealization of the $V(J)$ characteristic of an MJ SC results in it having a single-junction (SJ) form (4). Therefore, all the relations observed for an SJ SC are also valid for an idealized MJ SC.

In the open-circuit mode ($J = 0$, $V = V_{oc}$), we have

$$V_{oc} = E\ln\left(\frac{J_g}{J_s}\right). \quad (A.1)$$

The operating voltage V_m is expressed in terms of V_{oc} as [5]:

$$V_{oc} = V_{m0} + E\ln\left(1 + \frac{V_{m0}}{E}\right) \quad (A.2)$$

(the subscript “0” designates the zero-resistance case).

In addition, from (4) follows the relationship between the operating voltage V_{m0} and the operating current J_{m0}:

$$V_{m0} = E\ln\left(\frac{J_g - J_{m0}}{j_s}\right). \quad (A.3)$$

From (A.1), (A.2), and (A.3) follows the relations between the currents:

$$(J_g - J_{m0}) = \frac{J_{m0}E}{V_{m0}} = \frac{J_g E/V_{m0}}{1 + E/V_{m0}}. \quad (A.4)$$

Using (A.4) and (A.3), we can express V_{m0} in terms of J_g or J_{m0}:

$$V_{m0} = E\ln\left(\frac{J_g}{J_s(1 + V_{m0}/E)}\right) = E\ln\left(\frac{J_{m0}}{J_s V_{m0}/E}\right). \quad (A.5)$$

It is noteworthy that the dependences of V_m on J_g and J_{m0}, written in the implicit form (A.5), are almost logarithmic, and the dependence of V_{m0} on $(J_g - J_{m0})$ (A.3) is strictly logarithmic. In addition, in the absence of illumination ($J_g = 0$), the dark characteristic $V(J)$ has the form

$$V = E\ln\left(\frac{-J}{J_s}\right), \quad (A.6)$$

where $J < 0$.

Thus, the characteristic $V_{oc}(J_g)$ (A.1) coincides with $V_{m0}(J_g - J_{m0})$ (A.3) and the dark characteristic $V(J)$ (A.6). In the resistive case ($R_s \neq 0$), the series resistance affects the characteristics $V(J)$ (dark) and $V_{m0}(J_g - J_{m0})$, but has no effect on $V_{oc}(J_g)$. This makes it possible to interpret the characteristic $V_{oc}(J_g)$ as a zero-resistance characteristic $V_m(J_g - J_m)$ and that of the dark type $V(J)$, and this circumstance was used in Section 3.2.

2. DIFFERENTIAL RELATIONS

In Section 3.1, we used a set of derivatives $dV_{m0}/d(J_g - J_{m0})$, dV_{m0}/dJ_g, dV_{m0}/dJ_{m0}, related to the position of the maximum in V_m, From (A.3) follows, with (A.4) taken into account, that:

$$\frac{dV_{m0}}{d(J_g - J_{m0})} = \frac{E}{J_g - J_{m0}} = \frac{E}{J_{m0}(E/V_{m0})} = \frac{E(1 + E/V_{m0})}{J_g(E/V_{m0})}. \quad (A.7a)$$

To find the rest of the derivatives, we first rewrite the implicit function (A.5) as a difference of logarithms and then differentiate the result term-by-term and combine terms containing a derivative:

$$\frac{dV_{m0}}{dJ_g} = \frac{E}{J_{m0}(1 + 2E/V_{m0})} = \frac{E(1 + E/V_{m0})}{J_g(1 + 2E/V_{m0})}, \quad (A.7b)$$

$$\frac{dV_{m0}}{dJ_{m0}} = \frac{E}{J_{m0}(1 + E/V_{m0})} = \frac{E}{J_g}. \quad (A.7c)$$

The differential relations between the currents follow from (A.7a), (A.7b), and (A.7c):

$$\frac{dJ_{m0}}{dJ_g} = \frac{(1 + E/V_{m0})}{1 + 2E/V_{m0}}, \quad (A.8a)$$

$$\frac{d(J_g - J_{m0})}{dJ_g} = \frac{E/V_{m0}}{1 + 2E/V_{m0}}, \quad (A.8b)$$

$$\frac{d(J_g - J_{m0})}{dJ_{m0}} = \frac{E/V_{m0}}{1 + E/V_{m0}}. \quad (A.8c)$$

These derivatives are nonzero, which are used in Section 3.1.

ACKNOWLEDGMENTS

The authors are grateful to Prof. V.M. Andreev for his interest in, and attention to the study and to Prof. V.D. Rumyantsev for helpful discussions.

The study was supported in part by the Russian Foundation for Basic Research (grant no. 11-08-00539-a) and the Ministry of Education and Science of the Russian Federation (grant no. 16.51.11.6053).

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Translated by M. Tagirdzhanov