Determination of Series Resistance of Multijunction Solar Cells by Using Their Photoelectric Characteristic

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ABSTRACT: A new approach for the analysis of the IV curves of multijunction SC has been developed. It allowed obtaining analytical expressions taking into account the photogenerated current mismatch (disbalance) in the different subcells of such devices. It has been shown using these expressions that the series resistance can be represented as linear one up to achieving the maximum of efficiency (η) and does not depend on the radiation intensity (photogenerated current Jp, sunlight concentration ratio X).

The resistive losses in a concentrator SC results in appearance of a maximum on the dependence of the efficiency on X. For the same reason, there is a maximum on the dependence of the operation voltage on the photogenerated current characteristic (Vm(J)), such a behavior of the characteristics is used for determining the value of the series resistance R<sub>s</sub>. It has been justified analytically that such a resistance may be determined by the formula R<sub>s</sub> = (E/J<sub>p</sub>)ηmax, where E = AKT/q, A, J<sub>p</sub> are the local values of the ideality coefficient and photogenerated current at the maximum η (or V<sub>m</sub>).

It has been also justified that the value of R<sub>s</sub> determined by this formula does not depend on the spectral content of incident radiation, which is very useful for the express analysis. This has been experimentally shown for the triplejunction InGaP/GaAs/Ge SC.

Keywords: Multijunction Solar Cells, Series Resistance, Concentrator Cells

1 INTRODUCTION

One of the important factors affecting the photovoltaic conversion efficiency (η) of concentrator multijunction (MJ) solar cells (SCs) of different type are internal resistive losses specified by a term "series resistance". Due to the presence of this resistance, the photovoltaic conversion efficiency rise in increasing the illumination intensity (sunlight concentration ratio, X) changes to the decay portion forming a maximum [1–3]. The operating voltage (that in the optimum load point), V<sub>m</sub>, also reaches the maximum value [4].

Both the transverse resistance of SC layers and the lateral spreading resistance in the top layer contribute to the resistive losses [5–7]. There exist at least two main methods for analyzing the series resistance. The first one is simulation of a multicell equivalent circuit [5] or combination of cells [8–10]. The second one – representation of a SC, as an electrical circuit of elements connected in series: the generating part (comprised of photovoltaic (PV) p-n junctions) and the resistive one (lumped equivalent of resistive losses) [11]. When the contribution of the spreading resistance dominates, the lumped equivalent is a nonlinear resistance depending on the illumination intensity. In the work [5], an approach to the analysis of a multicell model, which allows describing such a behaviour and also determining main properties of the lumped series resistance has been proposed.

In the present work, a representation about that, in practically important cases (up to the efficiency maximum), such a lumped equivalent can be replaced by a fixed (linear and independent of the illumination intensity) series resistance R<sub>s</sub> has been suggested. In the work, a method for determining such a series resistance in a MJ SC, in both photogenerated current matched and mismatched ones have been elaborated.

A basic formula for determining the series resistance has been obtained analytically with the use of the presence of the operating voltage maximum and extended to the efficiency maximum. Also an experimental justification of the obtained analytical results, as applied to the MJ SCs based on the GaInP/Ga(In)As/Ge structure, is presented. As is known [1, 2, 12], the most effective at present solar cells have been created.

2 CURRENT – VOLTAGE CHARACTERISTIC OF A MJ SC – GENERAL, REDUCED AND IDEALIZED VIEW

Both current-voltage (I-V) characteristic and any PV characteristic of a MJ SC can be approximated by a set of segments [13]. On each segment, the V(J) characteristic of the generating part is formed by means of summing the voltages on the photovoltaic p-n junctions. These voltages depend on the dominating current passage mechanism (diffusion or recombination) in a definite segment. As a result, the V(J) characteristic of the segment (with allowing for re-
ractive losses simulated by the linear resistance connected in series) has a form:

\[ V = kT/q \sum_{i=1}^{n} \ln \left[ \frac{J_{g_i} - J}{J_{s_i}} \right] A_i - J \cdot R_s = \]

\[ kT/q \cdot \ln \left( \prod_{i=1}^{n} \left[ \frac{J_{g_i} - J}{J_{s_i}} \right] A_i \right) - J \cdot R_s, \quad (1) \]

where \( i \) - subcell number, \( J_{s_i} \) - preexponents (“saturation” currents), \( A_i \) - diode coefficients (idealties), which are equal to 1 (the diffusion mechanism is dominating) or to 2 (recombination mechanism), \( J_{g_i} \) - photogenerated currents, \( n \) - the number of subcells, \( R_s \) - series resistance.

If the smallest photogenerated current is symbolized as \( J_g \) (i.e. \( J_g = \min\{J_{g1}, J_{g2}, \ldots, J_{gn}\} \)), (1) can be presented in a form of a sum of three items:

\[ V = E \cdot \ln \left( \frac{J_g - J}{J_s} \right) - J \cdot R_s + V_a. \quad (2) \]

where the voltaic diode coefficient in the first item \( E = A \cdot kT/q \), \( A = \sum A_i \), \( J_s = \sqrt{n \prod_{i=1}^{n} J_{s_i} A_i} \).

The second item is resistive one. The third item allows for the mismatch of the photogenerated currents:

\[ V_a = kT/q \cdot \ln \left( \prod_{i=1}^{n} \left[ \frac{J_{s_i} \cdot J_g - J}{J_{s_i}} \right] A_i \right), \quad (3) \]

where \( \kappa_i = \frac{J_{s_i}}{J_{s_i}} \geq 1 \) are current mismatch coefficients.

Note that, on practice, the smallest photogenerated current \( J_g = \min\{J_{g1}, J_{g2}, \ldots, J_{gn}\} \) is equal to the short circuit current, since the \( J_{sc} < V_{oc}/R_s \) condition is fulfilled.

In a complete matching (all \( \kappa_i = 1 \)) the voltage \( V_a \) becomes to be zero, and the \( V(J) \) characteristic (2) takes the same form as in single-junction SC [5]. At the increase of the mismatch (rise of \( \kappa \)), \( V_a \) increases. Thus, the first item of the formula (2) is in current match (singlejunction) form, the second one is a usual voltage on the series resistance, and the value of the third item \( (V_a) \) characterizes the mismatch of the photogenerated currents.

As was pointed out above, the series resistance is the reason for formation of the maximum on the \( \eta(X) \) and \( V_m(X) \) characteristics. These maxima appear to be correlated. However, analytical description of the maximum is more convenient to perform using the following dependences of the operating voltage: \( V_m(J_g), \)

\( V_m(J_g), \) \( V_m(J_g - J_m). \) In this case, the following idealization is applied: the complete matching of the photogenerated currents \( (V_a = 0) \) and absence of resistance \( (R_s = 0) \). Than (2) takes the form:

\[ V = E \cdot \ln \left( \frac{J_g - J}{J_s} \right) \quad (4) \]

3 CURRENT MATCHED MJ SC

3.1 Approximate allowance for the series resistance

Suppose that in the case of balance \( (V_a = 0 \) in the expression (2) ), when the series resistance effect on the operating point \( (V_m, J_m) \) is taken into account, the following condition is fulfilled (Fig. 1):

\[ J_m \approx J_{m0} \quad (5) \]

(index “0” here and further means the resistanceless case).

Then the formula (2) for the operating point takes a form:

\[ V_m = V_{m0} - J_m \cdot R_s, \quad (6) \]

![Figure 1: Light (load) \( V(J) \) characteristic and position of the operating point 1 - with allowing for \( R_s \); 2 - without allowing for \( R_s \)]

As is shown in [14], differentiation (6) with allowing for (5) gives:

\[ \frac{dV_m}{dJ_m} = \frac{dV_{m0}}{dJ_{m0}} - R_s = \frac{E}{J_g} - R_s. \quad (7) \]

From the condition \( \frac{dV_m}{dJ_m} = 0 \), the basic correlation follows, which is central to the photo voltaic method for determining the series resistance (linear lumped equivalent of resistance):

\[ R_s = \frac{E_L}{J_{gL}}, \quad (8) \]
where $J_{gL}$ is the value of the photogenerated current $J_g$, at which the maximum value of the operating voltage $V_m$ is achieved. $E_L$ is the local voltage diode coefficient corresponding to the given voltage.

Thus, the main principle of $R_s$ determination is to find a value of the photogenerated current $J_{gL}$ at which the maximum $V_m$ is observed and the value of the local diode coefficient $E_L$ in this point.

### 3.2 Determination of series resistance

To find out the local diode coefficient, one may invoke two characteristics $V_{oc}(J_g)$ and $V_m(J_g - J_m)$, the $V_{oc}$ characteristic being interpreted as the resistanceless one $V_m(J_g - J_m)$ [14]. The required value $E_L$, correspondingly, is equal to the logarithmic slope $(\frac{\Delta V_{oc}}{\Delta \log J})$ in the $V_m$ maximum point (point A, 2).

![Figure 2: Mutual location of photovoltaic characteristics in a current matched MJ SC. 1 - $V_{oc}(J_g)$; 2 - $V_m(J_g - J_m)$, 3 - $V_m(J_g)$. $V_{oc}(J_g)$ and $V_m(J_g - J_m)$ characteristics coincide on the resistanceless portion](image)

Fig.2 illustrates the mentioned above method for finding the value of the series resistance on the base of the expression (8) at invoking $V_{oc}(J_g)$, $V_m(J_g)$ and $V_m(J_g - J_m)$ dependences. The value of $J_{gL}$ is equal to the current value in the point $L$, and the value of $E_L$ is determined from the slope of the portion in the region of point $A$ of the $V_{oc}(J_g)$ characteristic.

As is seen from the method for obtaining the basic formula (8) and from the procedure of finding the values $J_{gL}$ and $E_L$ (Fig.2), the proposed method for determining the series resistance is quite valid for singlejunction and current matched MJ SCs. This method can be generalized for the case of mismatched MJ SCs. This generalization is grounded on the allowance of properties of mismatch correction $V_a$ (3), which was equal to zero in the match case (p.3).

Thus, determination of the series resistance $R_s$ includes the steps: determination of $J_{gL}$ from the $V_{oc}(J_g)$ characteristic, determination of $E_L$ from correlation of the $V_m(J_g - J_m)$ and $V_{oc}(J_g)$ characteristics and application of the basic formula (8).

### 4 CURRENT MISMATCHED MJ SC

The method can be also applied to a current mismatched MJ SC. At a photogenerated currents mismatch as distinct from the case of matched photocurrents (p.3), the mismatch voltage $V_a$ (3) should be taken into account. Since the dependencies $V_m$ and $V_{oc}$ (p. 3.2) are used for determining the value $R_s$ by formula (8), it is necessary to take into account current mismatch corrections $(V_a, V_{a,oc})$.

Presence of the mismatch voltage and its dependence on current is explained in Fig.3 on the example of a Ge subcell. On the left, two cases of its light I-V characteristics are presented. In the match case, the Ge subcell photogenerated current is attenuated down to the minimum one of three subcells $J_g^{3J}$. In the mismatch (real) case, the photogenerated current exceeds the minimum one, $J_g^{Ge} > J_g^{3J}$. On the right, the result of subtraction of voltages, i.e. mismatch correction, is shown. It depends on current, which is presented in the figure in a normalized form, $\alpha = J_g^{3J}/J_g^{Ge}$. The mismatch voltage for the Ge subcell, with allowing for the introduced designations, has the form:

$$V_a^{Ge} = kT/q \cdot \ln \left( \frac{\kappa^{Ge}-1}{\alpha-1} \right)^{AGe}$$

which is a particular case of the formula (3).

![Figure 3: I-V characteristics of a Ge subcell in the match (bal) and (disb) mismatch cases (left) and the result of of subtracting of voltages (right).](image)
It has been shown in [14] that, in the operating mode \( (J = J_m, V = V_m) \), the mismatch correction to the operating voltage can be approximated as a constant, since \( \alpha_m = J_m / J_g^3 \approx const \). In the open circuit mode \( (J = 0, V = V_{oc}) \) this correction is strictly constant, since \( \alpha_{oc} = 0 \). For this reason, the basic formula (8) and the procedure for determining the series resistance remain to be the same as in the match case.

5 EXPERIMENTAL REALIZATION OF THE METHOD

As has been shown (p. 4), deduction of the basic formula (8) at applied approximations \( J_m \approx J_{m0} \) (5) and \( V_{oc} = const \) is one and the same for both current matched and current mismatched MJ SCs, and it does not depend on the mismatch correction \( V_{oc} \). Hence, the result of the series resistance determination should not be dependent of the irradiation spectral content. It can be practically useful for solving the problem of \( R_s \) determination at experimental study of MJ SCs.

The investigation was concerned with a triple-junction InGaP/GaAs/Ge SC fabricated by the MOCVD technique [15].

Several sets of \( V(J) \) characteristics have been obtained at different spectral content of incident radiation, and the \( V_m(J_g), V_m(J_g - J_m), V_{oc}(J_g) \) photovoltaic characteristics have been plotted (Fig.4).

The measurements were carried out at room temperature with using a simulator based on a pulsed xenon lamp with correcting light filters [16]. The measured short circuit current values were considered to be equal to the photogenerated currents \( (J_g) \) (even at a maximum lamp intensity). Four types of spectrum A, B, C, D have been used. Spectrum A – standard AM 1.5D, spectrum B – that of a nonfiltered xenon pulsed lamp, spectrum C – with a red light filter KG-2, enriched with blue light (300 - 650 nm), spectrum D – with a blue light filter RG-8 – enriched with red light (650 - 1000 nm).

The maxima of \( V_m(J_g) \) dependencies are located approximately at the same value of the photogenerated current \( (J_g)_L \) - Table 1; lines 1 – Fig.4). Location of the maxima of the \( V_m(J_g - J_m) \) characteristics is somewhat different \( (J_g)_A \) - Table 1; lines 2 – Fig.4). However, the corresponding values of the slope \( \Delta V_{oc} / 2.3 \Delta J_g(J_g) = E_L \) on the \( V_{oc}(J_g) \) characteristic are practically similar \( (E_L) \) - Table 1; lines 3 – Fig.4).

The observed position of the \( V_m(J_g) \) characteristics’ maxima correlates with done approximations. Some distinction between positions of the \( V_m(J_g - J_m) \) characteristics’ maxima does not affect the result of the \( R_s \) determination (Table 1). It is seen from Table 1 that the series resistance is equal to \( (14.0 \pm 0.3) \times 10^{-3} \text{Ohm} \cdot \text{cm}^2 \), i.e. the relative error off the method is 2%.

Figure 4: Experimental PV characteristics: 1 - \( V_m(J_g) \), 2 - \( V_m(J_g - J_m) \), 3 - \( V_{oc}(J_g) \) at different illumination spectra (A, B, C, D).

Table 1: Results of determination of the series resistance of the studied specimen at different spectra of incident irradiation.

<table>
<thead>
<tr>
<th>Spectrum</th>
<th>( J_g)_L ) A/\text{cm}^2</th>
<th>( J_g)_A ) A/\text{cm}^2</th>
<th>( E_L ) V</th>
<th>( R_s ) Ohm \cdot \text{cm}^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6.76</td>
<td>0.26</td>
<td>0.002</td>
<td>13.6E-3</td>
</tr>
<tr>
<td>B</td>
<td>6.78</td>
<td>0.092</td>
<td>0.007</td>
<td>14.3E-3</td>
</tr>
<tr>
<td>C</td>
<td>6.70</td>
<td>0.095</td>
<td>0.002</td>
<td>13.8E-3</td>
</tr>
<tr>
<td>D</td>
<td>6.75</td>
<td>0.096</td>
<td>0.006</td>
<td>14.2E-3</td>
</tr>
</tbody>
</table>

It follows from the results obtained that the elaborated PV method is applicable to MJ SCs, a result of the series resistance determination being not dependent of the incident irradiation spectrum content. Both the standard spectra (AM0, AM1.5 and others) and those distinct from standard ones can be used.

6 CONCLUSION

A new procedure for determining the lumped series resistance characterizing resistive losses in
practically valuable illumination intensity range up to the maximum of the efficiency or operating voltage have been elaborated.

An expression for experimental determination of the series resistance have been obtained analytically: $R_s = \frac{E_L}{J_g}$, where $E = A kT/q$ and $A$, $J_g$ are, respectively, the local ideality coefficient and the photogenerated current at such an illumination intensity, when the efficiency ($\eta$) or the operating voltage ($V_m$) are maximal ones. The value $E_L$ is determined from the local slope of the $V_{oc}(J_g)$ characteristic.

In applying the elaborated method, one can use both a standard (AMO, AM1.5 and others) spectral composition of the incident radiation and a nonstandard one. The series resistance of a triplejunction SC grown by the MOCVD technique has been obtained at different spectra. The obtained series resistance values were, practically, alike (with relative error of about 2%). It has been shown that the proposed method is applicable for analyzing experimental results obtained in investigating the I-V characteristics of MJ SCs illuminated by radiation of different spectral composition.

The PV method, in which presence of the maxima of $\eta$ or $V_m$ is used, is also appropriate for singlejunction SCs and, probably, for such ones, the load I-V characteristic of which can be approximated by an exponent.

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