

**SEGMENTAL CHARACTER, PECULIARITIES AND SIMILARITY OF THE DEPENDENCES:  
“EFFICIENCY – SUNLIGHT CONCENTRATION RATIO” AND  
“VOLTAGE – DARK CURRENT” IN MULTIJUNCTION SOLAR CELLS**

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**ABSTRACT:** Correlation and similarity of main characteristics of multijunction solar cells (MJ SC): “voltage – dark forward current density”,  $V$ - $J$ ; “efficiency (proportional to the introduced auxiliary term – “efficiency” voltage,  $V_\eta$ ) – sunlight concentration ratio (proportional to the photogenerated current density)”,  $\eta$  –  $C$  have been considered. It has been shown that  $V$ - $J$  and those characteristics have a shape of a continuous function consisting of segments. For each segment, an analytical dependence is deduced expressing the photogenerated current density through auxiliary definition – “efficiency” voltage  $V_\eta$ , which is proportional to the efficiency  $\eta$ . The segments of the analytical  $J_g - V_\eta$  characteristic contain the same two parameters as the corresponding segments of the initial dark  $J$ - $V$  characteristic do – the preexponential factor  $J_0$  and the diode coefficient  $A$ . Hence, the  $J_g - V_\eta$  characteristic is uniquely associated (at idealization of the connecting part – tunneling diodes in MJ SCs) with the initial dark  $J$ - $V$  characteristic. The equivalent electrical circuit of a MJ SC is considered and adjusted to an electrical circuit of a “virtual” singlejunction SC. Determined were the preexponential factors  $J_0$  and the diode coefficients  $A$  for each segment of the  $J$ - $V$  characteristic. For them, electrically idealized  $\eta$  -  $C$  characteristics were calculated and plotted. The influence of the connecting elements (tunneling diodes) has been simulated and taken into account by selecting an ohmic series resistance  $R_S$ . A good fit of the calculated  $\eta$  -  $C$  dependencies to experimental ones for InGaP/GaAs/Ge MJ SCs has been obtained. An abrupt increase in the current density has been observed and analyzed in the experimental “voltage – dark forward current density” characteristics of MJ SCs.

**Keywords:** Multijunction Solar Cells, Solar Cell Efficiencies, Concentrator Cells

## 1 INTRODUCTION

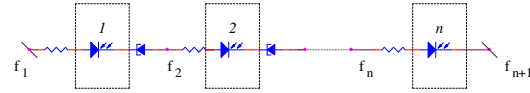
One of the most important SC characteristics is the dark “forward current density – voltage” characteristic,  $J$ - $V_\phi$ . Actually, just the  $J$ - $V_\phi$  characteristic determines the potential efficiency of a SC based on realistic photovoltaic (PV)  $p$ - $n$  junctions [1]. The potential, i.e. limiting, efficiency, which is theoretically feasible at the condition that the realistic PV  $p$ - $n$  junctions are complimented with idealized optical and electrical (connecting) parts of a SC. Such an idealization means: first, the external quantum yield of PV  $p$ - $n$  junctions forming a MJ SC is equal to unity ( $\gamma = 1$ ); second, the voltage on the connecting part is equal to zero ( $V_C = 0$ ) of a MJ SC comprising tunneling  $p^{++}$ - $n^{++}$  junctions (TDs) and ohmic series resistance of the whole structure  $R_S$ . The zero voltage on the MJ SC connecting part means also that all connecting components are photopassive (absence of a photo-electromotive force).

The aim of the present work is to show the correlation between two MJ SC characteristics: initial “dark current density – voltage”  $J$ - $V_\phi$  and photovoltaic “efficiency – sunlight concentration ratio”  $\eta$  -  $C$ . In using the auxiliary definition “efficiency” voltage  $V_\eta$ , which is proportional to the efficiency  $\eta$ , an unambiguous link is established between the dark  $J$ - $V_\phi$  and photovoltaic  $\eta$  -  $J_g$  characteristics at the condition that all photovoltaic  $p$ - $n$  junctions are photocurrent matched or close to that. Determination of segments and other peculiarities in the MJ SC experimental dependencies – “voltage – dark forward current density” and “efficiency – generation current density” – is accessible and useful method for testing and determining the parameters influencing on the SC efficiency.

The SC structures were grown by the MOCVD technique.

## 2 EQUIVALENT ELECTRICAL CIRCUIT OF A MJ SC

A MJ SC consists of several PV and TD  $p$ - $n$  junctions connected in series by resistors (Fig. 1).



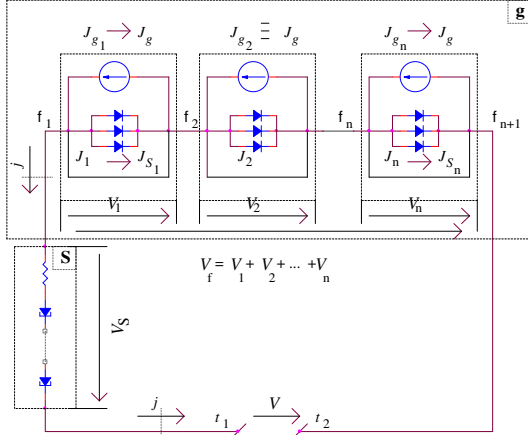
**Figure 1:** Schematic of the MJ SC electrical circuit.

The MJ SC circuit is obtained by separating-out the PV junctions into a photovoltaic part and the TDs and series ohmic resistors of the structure into another one. After this, the equivalent circuit consists of a pair of two-terminal networks (generating and connecting) connected in series. The generating MJ SC part consists of PV  $p$ - $n$  junctions connected in series, and the connecting part – of TDs connected in series and the structure ohmic resistance  $R_S$  (Fig. 2). In turn, the equivalent circuit of each PV  $p$ - $n$  junction is represented by an ideal current source (characterized by the generation current  $J_g$ ) and three idealized diodes connected in parallel, each of which is characterized by the exponential “current density – voltage” dependence,  $J$ - $V$ .

$$J_{ki} = J_{0ki} \left( \exp \frac{V_k}{A_{ki} \varepsilon} - 1 \right), \quad (1)$$

where  $\varepsilon \equiv kT/q$ ,  $A_{ki}$  is the diode coefficient (ideality coefficient);  $J_{0ki}$  is the preexponential factor or so-called the “saturation” current;  $k$  is the first lower index of the diode coefficient and of the MJ SC photovoltaic  $p$ - $n$  junction factor, for example, for a triplejunction SC,  $k = 1$  (lower), 2 (middle), ..., and  $n$  (upper);  $i \equiv d, r, t$  is the

second lower index indicating one of three possible current flow mechanisms in each PV  $p-n$  junction forming a MJ SC: diffusion (Shockley,  $i = d$ ,  $A_{kd} = 1$ ) [2]; recombination (Sah – Noyce – Shockley,  $i = r$ ,  $A_{kr} = 2$ ) [3]; tunneling of the defect – depended character (Esaki,  $i = t$ ,  $A_{kt} > 2$ ), which gives so-called “excess” or multistep current component [4].



**Figure 2:** Principal intermediate MJ SC electrical circuit consisting of a couple of two-terminal networks: generating –  $g$  and connecting –  $S$ .

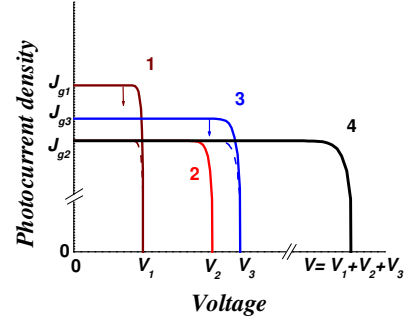
Thus, the “internal” current through each PV  $p-n$  junction ( $J_k = J_1, J_2, \dots, J_n$ ) is a sum of three exponential components: diffusion, recombination and tunneling:  $J_k = J_{kd} + J_{kr} + J_{kt}$ ,  $k = 1, 2, \dots, n$ .

In the general case, the generation current in a MJ SC is determined by the least of the generated currents in some PV junction of those comprising a MJ SC. For example, in a triplejunction InGaP/GaAs/Ge SC, the smallest photogenerated current is, most often, in the middle GaAs  $p-n$  junction (Fig.2, 3). For this reason, the currents generated in the top InGaP and bottom Ge  $p-n$  junctions are limited (forced for matching) by the generation current value in the GaAs  $p-n$  junction (Fig. 2, 3). In this case we may suppose that the generation current  $J_{g2} \equiv J_g$ , that is, to fulfill the condition of “complete” matching, it is necessary to make substitutions  $J_{S1} \equiv J_{01}(J_g/J_{g1})$  and  $J_{S3} \equiv J_{03}(J_g/J_{g3})$ .

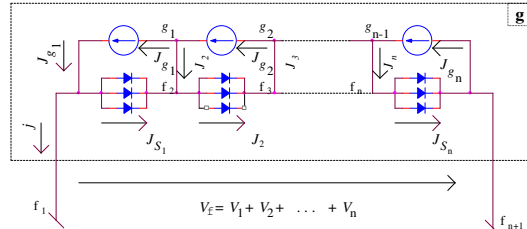
At the condition of generation current matching in a MJ SC, when, for example, two of ideal current sources are equivalent  $J_{g1} = J_{g2}$ , and the potentials in sites  $g_1$  and  $f_2$  (Fig. 4) are equal to each other, the corresponding current  $J_2$  does not flow in the circuit portion ( $g_1 - f_2$ ), i.e.  $J_2 = J_{g2} - J_{g1} = 0$ . It means that this branch in the equivalent circuit may be not accounted for. In this case, these two ideal current sources connected in series ( $J_{g1} = J_{g2}$ ) may be replaced by one ideal current source. Applying similar approach for all  $p-n$  junctions comprising a MJ SC we obtain the generating part in a form of a two-terminal network (Fig. 5) with a common generation current  $J_g$ .

Besides, the dark  $J-V_\phi$  characteristics of PV  $p-n$  junctions connected in series, for example, for the case of a triplejunction InGaP/GaAs/Ge SC, one may replace by a single “resulting” dark  $J-V_\phi$  characteristic of some “virtual” single  $p-n$  junction [1]. Then, the generating part in the MJ SC equivalent circuit will represent a resulting “virtual” PV  $p-n$  junction. In turn, it is

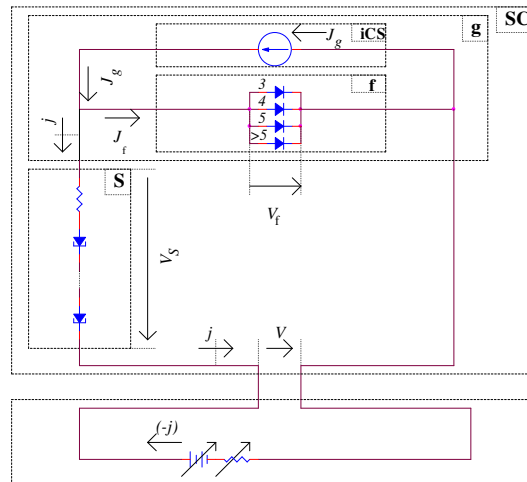
represented by an ideal current source (characterized by the generation current  $J_g$ ) and idealized diodes connected in parallel (Fig. 5), each of which is characterized by an exponential  $J-V_\phi$  dependence.



**Figure 3:** Schematic of the load  $J-V$  characteristics of PV  $p-n$  junctions comprising, for example, triplejunction InGaP/GaAs/Ge SC (in the case of photocurrent mismatching): 1 – bottom Ge  $p-n$  junction; 2 – middle GaAs  $p-n$  junction; 3 – top InGaP  $p-n$  junction; 4 – resulting load characteristic of triplejunction InGaP/GaAs/Ge SC in the case of forced current matching of the middle GaAs  $p-n$  junction. Conditions for forced matching:  $J_g \equiv J_{g2}$ ,  $J_{S3} = J_{03}(J_g/J_{g3})$  and  $J_{S1} = J_{01}(J_g/J_{g1})$  (Fig.2).



**Figure 4:** Intermediate electrical circuit of the generating part of a MJ SC.



**Figure 5:** Equivalent electrical circuit of a MJ SC:  $g$  is the generating part representing a “virtual” PV  $p-n$  junction (Fig. 3), which, in turn, is presented by an ideal current (characterized by the generation current,  $J_g$ ) and idealized diodes connected in parallel;  $S$  is the connecting part of TD and ohmic structure resistance  $R_S$  connected in series.

### 3 MAIN CHARACTERISTICS OF A MULTIJUNCTION SOLAR CELL

#### 3.1 Dark and light $J$ - $V$ characteristics of a photovoltaic $p$ - $n$ junction

The dark “forward current density – voltage” characteristic,  $J$ - $V_\varphi$ , of each PV  $p$ - $n$  junction comprising a MJ SC may be described by a sum of three current components [5]: diffusion (diode coefficient  $A_d = 1$ ), recombination ( $A_r = 2$ ) and tunneling-trapping ( $A_t > 2$ ) ones:

$$J = J_{0r} (\exp \frac{V_\varphi}{A_r \varepsilon} - 1) + J_{0t} (\exp \frac{V_\varphi}{A_t \varepsilon} - 1) + J_{0d} (\exp \frac{V_\varphi}{A_d \varepsilon} - 1) \quad (2)$$

where,  $V_\varphi$  is voltage on the space charge region (SCR) of a  $p$ - $n$  junction equal to the difference between the Fermi quasilevels on the interfaces and inside the SCR,  $J_{0r}$ ,  $J_{0t}$ ,  $J_{0d}$  are corresponding preexponential factors. Note that in some cases, in  $p$ - $n$  junctions based on  $A^{III}B^V$  compounds, also a current component was observed, for which the diode coefficient is a rational number,  $1 < A < 2$ .

The dark  $J$ - $V_\varphi$  characteristic (2) may be approximated by three segments, the dependence of voltage in the SCR on the dark current for each of which has a form [1]:

$$V_\varphi = A\varepsilon \cdot \ln \left( \frac{J}{J_0} + 1 \right) \quad (3)$$

At such a segmental approximation, the  $J$ - $V_\varphi$  characteristic in the whole is a continuous function comprised of separate segments. At  $V_\varphi > A_r \varepsilon$  or  $J \gg J_{0r}$ , one may neglect unity in the brackets in the expressions (2) and (3). Then the voltage and current boundaries between neighbouring segments are:

$$\begin{cases} V_{n(n+1)} = \frac{A_n A_{(n+1)}}{A_n - A_{(n+1)}} \varepsilon \cdot \ln \left( \frac{J_{0n}}{J_{0(n+1)}} \right) \\ J_{n(n+1)} = \frac{J_{0n}^{A_n / (A_n - A_{(n+1)})}}{J_{0(n+1)}^{A_{(n+1)} / (A_n - A_{(n+1)})}} \end{cases} \quad (4)$$

Where  $n$  and  $(n+1)$  are indices of neighbouring segments.

In particular, for the recombination – diffusion boundary:

$$V_{rd} = 2\varepsilon \ln \left( \frac{J_{0r}}{J_{0d}} \right); \quad J_{rd} = \frac{J_{0r}^2}{J_{0d}} \quad (5)$$

Smoothing-out between neighboring segments is fulfilled by summation of the current components (2).

The segment of the light “voltage – current density” characteristic is obtained from the corresponding segment of the dark one (3):

$$J = J_g - j, \quad (6)$$

where,  $J_g$  is the generation current density, which is proportional to the illumination intensity related, in particular, to the sunlight concentration ratio  $C$ ;  $j$  is the current density in the external circuit;  $J$  is the density of

the forward current through a  $p$ - $n$  junction. In (6), directions of the currents  $J_g$  and  $j$  are so chosen that they are positive on the load portion of the light characteristic. Substitution of (6) into (3) gives the segment of the light “voltage – current density” characteristic:

$$V_\varphi = A\varepsilon \ln \left( \frac{J_g - j}{J_0} + 1 \right) \quad (7)$$

#### 3.2 Light and dark $J$ - $V$ characteristics of the photovoltaic part of a MJ SC

The segment of the light “voltage – current density” characteristic of the photovoltaic part of a MJ SC (PV  $p$ - $n$  junctions connected in series) is obtained by summation of voltages of the segments (7) of the PV  $p$ - $n$  junctions:

$$V_\varphi = \sum_{i=1}^n V_{\varphi i} = \varepsilon \ln \left( \prod_{i=1}^n \left( \frac{J_{gi} - j}{J_{0i}} + 1 \right)^{A_i} \right) \quad (8)$$

where  $i$  is the PV  $p$ - $n$  junction number,  $n$  is the amount of PV  $p$ - $n$  junctions. At the condition of equality of the generation current densities ( $J_{g1} = J_{g2} = \dots = J_{gn} = J_g$  and neglecting unity in brackets of (8) (which is valid at  $V_{\varphi i} > A_i \varepsilon$ ) we shall obtain the segment characteristic:

$$\begin{cases} V_\varphi = A\varepsilon \ln \left( \frac{J_g - j}{J_0} \right) \\ j = J_g - J_0 \exp \left( \frac{V_\varphi}{A\varepsilon} \right) \end{cases} \quad (9)$$

where the diode coefficient  $A = A_1 + A_2 + \dots + A_n$  and the preexponential factor  $J_0 = \sqrt[A]{J_{01}^{A_1} \cdot J_{02}^{A_2} \cdot \dots \cdot J_{0n}^{A_n}}$ .

Note that the segment (9) has the same shape as the segment of a separate PV  $p$ - $n$  junction (7), but has resulting values of the diode coefficient and the preexponential factor.

The segment of the dark characteristic is obtained from (9) at a zero generation current  $J_g = 0$ , and according to (6)  $J = -j$ .

$$\begin{cases} V_\varphi = A\varepsilon \cdot \ln \left( \frac{J}{J_0} + 1 \right) \\ J = J_0 (\exp \left( \frac{V_\varphi}{A\varepsilon} \right) - 1) \end{cases} \quad (10)$$

Unity is added in the brackets for giving a standard from to the expressions (10). Note that the segment of the MJ SC dark characteristic has just the same shape as that of the segment of a separate PV  $p$ - $n$  junction (3). Thus, the whole dark  $J$ - $V$  characteristic of the photovoltaic part of a MJ SC has the same shape as the whole dark characteristic of a separate PV  $p$ - $n$  junction. That is it represents a continuous function comprised of segments. Smoothing-out between the segments takes place similar to (2) due to summation of the segmental currents (10), but only in that case, when the diode coefficients of the segments decrease with current. As is distinct from a separate PV  $p$ - $n$  junction, it is not always possible for a MJ SC.

In the case of open circuit  $j=0$ ,  $V=V_{oc}$ . Then it follows for the segment from (9):

$$J_g = J_0 \exp\left(\frac{V_{oc}}{AE}\right) \quad (11)$$

The expression (11) for the segment of the  $J_g$ - $V_{oc}$  characteristic coincides with the segment (9) for the dark characteristic that is it has the same values of the preexponential factor  $J_0$  and the diode coefficient  $A$ . Correspondingly, the whole  $J_g$ - $V_{oc}$  characteristic coincides with the dark one and also has a shape of a continuous function comprised of segments.

3.3 Correlation between the MJ SC efficiency and the generation current.

Voltage  $V_m$  in the optimum load point is determined from the condition of the maximum power generated on the external load [6]. For this, the power derivative is taken to zero  $\frac{d}{dV_\varphi}(j \cdot V_\varphi) = 0$ , where the segment current

is given by the formula (9). As a result, to determine  $V_m$ , it is necessary to find the root of the transcendental equation [2]:

$$\begin{cases} V_{oc} = V_m + E \ln\left(1 + \frac{V_m}{E}\right) \\ \left(1 + \frac{V_m}{E}\right) \exp\left(\frac{V_m}{E}\right) = \exp\left(\frac{V_{oc}}{E}\right) \end{cases} \quad (12)$$

where  $E \equiv AKT/q = A\varepsilon$ . In correspondence to (9)

$$J_m = J_g - J_0 \exp\left(\frac{V_m}{E}\right) \quad (13)$$

or substituting  $J_0$  from (11) into (13) and  $\exp\left(\frac{V_{oc}}{E}\right)$  from (12)

$$J_m = J_g \cdot V_m / (V_m + E) \quad (14)$$

Then the efficiency:

$$\eta = J_m V_m / P_{inc} = \frac{J_g}{P_{inc}} \cdot \frac{V_m^2}{V_m + E} = \frac{V_\eta}{V_{conv}} \quad (15)$$

where  $P_{inc}$  is the power density of radiation incident on the SC photoactive surface. At a fixed spectrum (AM0, AM1.5, monochromatic radiation and so on) the ratio  $\frac{P_{inc}}{J_g} = V_{conv}$  does not depend on the illumination intensity.

For this reason, the whole dependence of the efficiency on the generation current (proportional to the sunlight concentration ratio  $C$ ) is expressed through the “efficiency” voltage [1, 5]:

$$V_\eta \equiv \frac{V_m^2}{V_m + E} \quad (16)$$

With the use of this equation, the voltage  $V_m$  is

expressed through the “efficiency” voltage  $V_\eta \propto \eta$ :

$$V_m = \frac{1}{2} \left( V_\eta + \sqrt{V_\eta(V_\eta + 4E)} \right) \quad (17)$$

A similar correlation between  $V_\eta (\propto \eta)$  and  $J_g (\propto C)$  is deduced from the combination of (11), (12) and (17):

$$J_g = J_0 \left( 1 + \frac{V_\eta + \sqrt{V_\eta(V_\eta + 4E)}}{2E} \right) \exp\left(\frac{V_\eta + \sqrt{V_\eta(V_\eta + 4E)}}{2E}\right) \quad (18)$$

The whole  $J_g (\propto C) - V_\eta (\propto \eta)$  characteristic is comprised of the shape (18) segments and is a continuous function with bends, smoothing-out of which occurs as a result of summation of the segmental generation current.

3.4 Accounting for the effect of the connecting elements on the MJ SC efficiency.

The voltage  $V$  on the external load (Fig. 5) is less than the voltage generated by photovoltaic  $p$ - $n$  junctions  $V_\varphi$  by a value equal to that on the MJ SC connecting part  $V_S$ ;  $V = V_\varphi - V_S$ . At small sunlight concentration ratios ( $C < 10$ ), one may neglect this difference, if  $V_S < E$ .

To estimate the effect of the connecting part on the MJ SC efficiency, we suppose the following. First, the whole connecting part (including the tunneling  $p^{++}$ - $n^{++}$  junctions) is simulated by one series resistance  $R_S$ . Second, we consider that accounting for  $R_S$  does not practically change the operation current  $J_m$ , which, as follows from (14), is approximately equal to  $J_g$  at  $V_m \gg E$ . Only the operation voltage  $V_m$  and, according to (16), the “efficiency” voltage  $V_\eta$  decrease by the value  $\sim J_g R_S$ . Such a suggestion is based on that the shape of the resistanceless (not accounting for  $R_S$ ) light  $J$ - $V$  characteristic is close to rectangular one [1]. Therefore, the influence of  $R_S$  on the efficiency lies in the reduction of the resistanceless “efficiency” voltage by the value  $J_g R_S$ . In the analytical form, the  $\eta (\propto V_\eta) - C (\propto J_g)$  dependence is the following:

$$\begin{cases} J_g = J_0 \left( 1 + \frac{V_\eta + \sqrt{V_\eta(V_\eta + 4E)}}{2E} \right) \exp\left(\frac{V_\eta + \sqrt{V_\eta(V_\eta + 4E)}}{2E}\right) \\ V_\eta = V_\eta + J_g R_S \end{cases} \quad (19)$$

where  $V_{\eta 0}$  is the resistanceless “efficiency” voltage. Unlike form (18), the dependence (19) between  $J_g$  and  $V_\eta$  acquired an implicit form. Besides, a maximum appears at

$$J_{gm} \approx \frac{E}{R_S} = \frac{A\varepsilon}{R_S}$$

In the maximum point,  $\frac{d\eta}{dJ_g} = 0$ . It follows from (15)

and (19):

$$\frac{dV_\eta}{dJ_g} = \frac{dV_{\eta 0}}{dJ_g} - R_S \quad (20)$$

The derivative  $\frac{dV_{\eta 0}}{dJ_g} = \left( \frac{dJ_g}{dV_{\eta 0}} \right)^{-1}$  is obtained from the equation (19) simplified at  $V_\eta \gg 4E$  to:

$$J_g \approx 2.72 \cdot J_0 \cdot V_{\eta_0} / E \cdot \exp(V_{\eta_0} / E) \quad (21)$$

As is follows from (21), in using the inequality

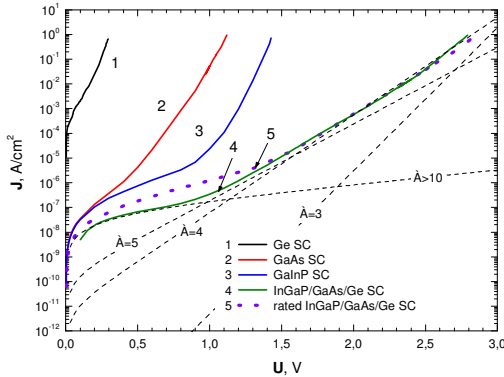
$$V_{\eta} \gg 4E, \quad \frac{dJ_g}{dV_{\eta_0}} \approx \frac{J_g}{E}.$$

As a result, it is clear from (20) that the generation current  $J_g$  in the efficiency maximum point  $J_{gm} \approx E/R_S$ .

## 4 RESULTS AND DISCUSSION

### 4.1 Dark forward $J$ - $V_{\phi}$ characteristics of singlejunction SCs based on the Ge, GaAs and GaInP $p$ - $n$ junctions and triplejunction InGaP/GaAs/Ge MJ SC

All experimental dark and photovoltaic characteristics were measured at room temperature. To plot the initial resistanceless dark characteristics by the laced method [5], the dark forward  $J$ - $V$  and  $J_{sc}$ - $V_{oc}$  ones were obtained in the forward current density range of  $(10^{-12} - 2) \text{ A/cm}^2$ . In GaAs and InGaP homo  $p$ - $n$  junctions, three components of the forward current were observed: diffusion ( $A = 1$ ), recombination ( $A = 2$ ) and tunneling-trapping (excess) ( $A > 2$ ) components (Fig. 6), [1]. The “excess” component reveals at the current densities lower than  $10^{-6} \text{ A/cm}^2$  for GaAs,  $5 \cdot 10^{-6} \text{ A/cm}^2$  for InGaP and  $5 \cdot 10^{-4} \text{ A/cm}^2$  for Ge. The values of the recombination and diffusion preexponential factors were: for the SCs based on GaAs homo  $p$ - $n$  junctions  $(1.5) \cdot 10^{-11} \text{ A/cm}^2$  and  $(1.5) \cdot 10^{-21} \text{ A/cm}^2$ ; for InGaP  $(1.7) \cdot 10^{-14} \text{ A/cm}^2$  and  $(1.5) \cdot 10^{-26} \text{ A/cm}^2$ . The diffusion preexponential factor in the investigated Ge  $p$ - $n$  junction was about  $1 \cdot 10^{-6} \text{ A/cm}^2$ .



**Figure 6:** Experimental dark  $J$ - $V$  characteristics laced with corresponding to them  $J_{sc}$ - $V_{oc}$  characteristics of singlejunction SCs: 1 – Ge, 2 – GaAs, 3 – InGaP; 4 – monolithic multijunction InGaP/GaAs/Ge with segments:  $A > 10$ ,  $J_{0(A>10)} = 6.0 \cdot 10^{-8} \text{ A/cm}^2$ ;  $A = 5$ ,  $J_{0(A=5)} = 1.5 \cdot 10^{-10} \text{ A/cm}^2$ ;  $A = 4$ ,  $J_{0(A=4)} = 7 \cdot 10^{-12} \text{ A/cm}^2$ ;  $A = 3$ ,  $J_{0(A=3)} = 8 \cdot 10^{-18} \text{ A/cm}^2$ . 5 – rated  $J$ - $V_{\phi}$  characteristic of a “virtual” InGaP/GaAs/Ge  $p$ - $n$  junction obtained from the characteristics of the Ge, GaAs and InGaP SCs.

Fig. 6 presents the resistanceless  $J$ - $V_{\phi}$  characteristic of the GaInP/GaAs/Ge SC and the rated “virtual” one for the similar triplejunction SC. The resistanceless  $J$ - $V_{\phi}$  characteristic of the Ge, GaAs, InGaP and GaInP/GaAs/Ge SC was obtained by the laced method from the  $J$ - $V$  and  $J_{sc}$ - $V_{oc}$  characteristics [5]. In the presented characteristic of InGaP/GaAs/Ge MJ SC, the

following segments were observed: tunneling-trapping with  $A > 10$ , the preexponential factor of  $6 \cdot 10^{-8} \text{ A/cm}^2$ , two recombination – diffusion with  $A = 5$  and  $A = 4$ , the preexponential factors of  $1.5 \cdot 10^{-10} \text{ A/cm}^2$  и  $7 \cdot 10^{-12} \text{ A/cm}^2$  and diffusion with  $A = 3$ , the preexponential factor of  $8 \cdot 10^{-18} \text{ A/cm}^2$ .

The rated  $J$ - $V_{\phi}$  characteristic (curve 5 on Fig.6) of the “virtual” triplejunction SC is plotted by the method of summation of voltages at one and the same current value from those presented on the  $J$ - $V_{\phi}$  characteristic plots for singlejunction Ge, GaAs and GaInP SCs. As is seen from the plots, the “virtual” dark  $J$ - $V_{\phi}$  (GaInP/GaAs/Ge) characteristic fits quite well to the experimental  $J$ - $V_{\phi}$  one of the monolithic GaInP/GaAs/Ge SC. The difference between the experimental and “virtual” characteristics at the current densities  $< 10^{-6} \text{ A/cm}^2$  may be explained by the difference in technologies for fabricating the specimens for investigation and indicates a definite effect of the postgrowth technology on the excess component of the current in a SC. The discrepancy in the current density region higher than  $100 \text{ mA/cm}^2$  indicates the rising effect of the connecting elements (tunneling diodes) in the monolithic GaInP/GaAs/Ge MJ SC.

### 4.2 Abrupt rise in current on the dark $J$ - $V_{\phi}$ characteristic of a MJ SC

On the dark  $J$ - $V$  characteristic of the MJ SC, a sharp rise of the forward current has been observed in the region of the current densities  $< 10^{-7} \text{ A/cm}^2$  and voltage  $< 0.6 \text{ V}$  (Fig. 7). This effect was not observed in recording the  $J$ - $V$  characteristic in the reverse direction. The proposed interpretation of this effect is based on the similarity of GaAs and InGaP PV  $p$ - $n$  junctions connected by a tunneling diode in a MJ SC and a dynistor. The structural circuit of a dynistor is a four-layer  $p$ - $n$ - $p$  structure, which can be represented in a form of two  $p$ - $n$ - $p$  and  $n$ - $p$ - $n$  transistors with current transfer coefficients  $h_{21B1}$  and  $h_{21B2}$ , the collector of which is common one, Fig. 8 [8]. At pointed out polarity of the external voltages, the emitter junctions of both transistors ( $\Pi_1$  and  $\Pi_3$ ) are connected in the forward direction and the common collector ( $\Pi_2$ ) – in opposite one. The current through the junction  $\Pi_2$  consists of the reverse current of this  $p$ - $n$  junction  $I_{KBO}$ , current of holes injected by the junction  $\Pi_1$  and came to  $\Pi_2$ , current of electrons injected by the junction  $\Pi_3$  and came to  $\Pi_2$ . Thus,

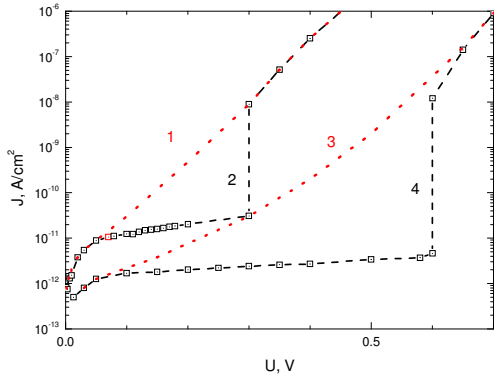
$$I = h_{21B1}I_1 + h_{21B2}I_3 + I_{KBO} \text{ and, hence,}$$

$$I = I_{KBO} / [1 - (h_{21B1} + h_{21B2})] = I_{KBO} / (1 - h_{21B}).$$

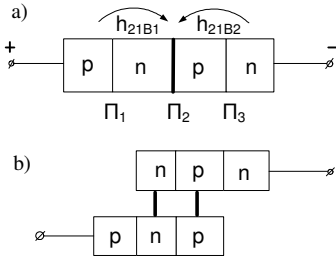
This formula is similar to the formula for the current passed though the transistor at the switched off base, but here

$$h_{21B} = h_{21B1} + h_{21B2}$$

is the total coefficient of the current transfer of both components of transistors. For this reason, in such a  $p$ - $n$ - $p$  structure, as in a transistor, there exists a positive reverse bias by current, which is the following. With increasing voltage,  $I_{KBO}$  and, hence, the current through the structure increase. The rise of the current results in the increase of  $h_{21B}$ , which, in turn, determines further the current rise. Apparently, just this effect is observed (Fig. 7) on the experimental dark  $J$ - $V_{\phi}$  characteristics for the investigated MJ SCs.



**Figure 7:** Dark forward  $J$ - $V$  characteristics of a MJ SC with a sharp rise of the forward current: 1, 3 – in lowering voltage values; 2, 4 – in raising voltage values.

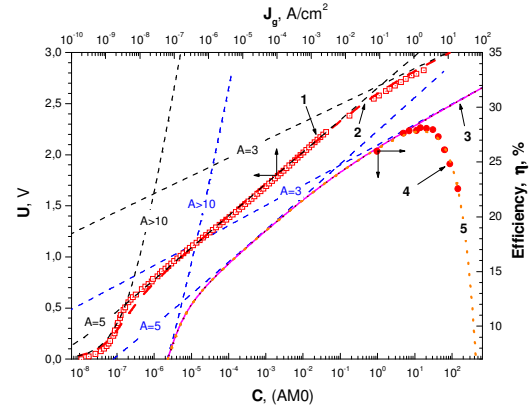


**Figure 8:** Schematic of a dynistor – a) and its representation in a form of an equivalent circuit of two transistors with a common collector – b).

4.3 Rated and experimental “efficiency – sunlight concentration ratio” characteristics  $\eta(\propto V_\eta) - C(\propto J_g)$  of InGaP/GaAs/Ge MJ SCs.

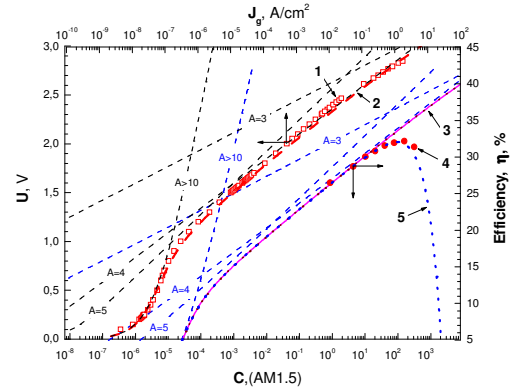
The experimental dark resistanceless  $J$ - $V_\phi$  characteristic (Fig. 6, 9, 10) gives a possibility to calculate by the formulae (20), (21) the  $J_g - V_\eta$  characteristic for each segment of the dark  $J$ - $V_\phi$  characteristic. As has been shown, this characteristic is proportional to the  $C(\propto J_g) - \eta(\propto V_\eta)$  characteristic.

The proportionality coefficient between  $J_g$  and  $C$  is the generation current at one sum ( $C=1$ )  $J_{g,c=1}$ , which is determined experimentally and coincides with the short circuit current of a MJ SC at  $J_{sc}R_S < V_{oc}$ . The proportionality coefficient between  $V_\eta$  and  $\eta$  is the conversion voltage  $V_{conv} = P_{inc} / J_g$ , which, at a fixed irradiation spectrum, does not depend on the illumination intensity and is determined at  $C=1$ ,  $V_{conv} = P_{inc,C=1} / J_{g,C=1}$ , ( $P_{inc,C=1} = 136.6 \text{ mW/cm}^2$ , AM0;  $100 \text{ mW/cm}^2$ , AM1.5). Summing the generation current of all segments,  $J_g = \sum_i J_{gi}$ , where  $i$  is the segment index, we obtain the complete  $\eta(\propto V_\eta) - C(\propto J_g)$  characteristic. Summation leads to smoothing-out the angles between the segments, if the diode coefficients of the dark  $J$ - $V_\phi$  characteristic decrease with the generation current.



**Figure 9:** Experimental and rated characteristics of a monolithic InGaP/GaAs/Ge SC:  $J - V_\phi$  characteristic, 1 – experiment, 2 – fitting;  $\eta(\propto V_\eta) - C(\propto J_g)$  characteristic, 3, 5 – calculation ( $R_S = 0$ ,  $R_S = 0.3 \text{ Ohm}\cdot\text{cm}^2$ ), 4 – experiment, (AM0,  $136 \text{ mW/cm}^2$ ). Segments:  $A > 10$ ,  $J_{0(A>10)} = 4.0 \cdot 10^{-9} \text{ A/cm}^2$ ;  $A = 5$ ,  $J_{0(A=5)} = 5 \cdot 10^{-13} \text{ A/cm}^2$ ;  $A = 3$ ,  $J_{0(A=3)} = 2.4 \cdot 10^{-18} \text{ A/cm}^2$ .

Fig. 9 and 10 present the experimental and rated  $J$ - $V_\phi$  and  $\eta - C$  characteristics for the InGaP/GaAs/Ge MJ SC in exciting by the AM0 and AM1.5 sunlight, respectively. The plotted  $\eta - C$  dependences calculated by the formulae (19), (20) for the AM0 and AM1.5 sunlight coincide well with the experimental ones at found by fitting  $R_S = 0.3 \text{ ohm}\cdot\text{cm}^2$  and  $0.075 \text{ ohm}\cdot\text{cm}^2$ , correspondingly. The coincidence indicates the validity of the suppositions and approximations in obtaining the equation (19) with allowing for (20): photovoltaic  $p$ - $n$  junctions are generation current matched or close by current ( $J_{g1} = J_{g2} = \dots = J_{gn} = J_g$ ; the whole connecting part of the MJ SC is simulated by an ohmic series resistance  $R_S$ ; to account for the effect of the series resistance on the MJ SC efficiency, the “efficiency” voltage  $V_\eta(\propto \eta)$  calculated in the resistanceless approximation ( $R_S = 0$ ) is decreased by the value  $J R_S$ .



**Figure 10:** Experimental and rated characteristics of a monolithic InGaP/GaAs/Ge SC:  $J - V_\phi$ , 1 – experiment, 2 – fitting;  $\eta(\propto V_\eta) - C(\propto J_g)$ , 3, 5 – calculation ( $R_S = 0$  and  $0.075 \text{ Ohm}\cdot\text{cm}^2$ ), 4 – experiment, (AM1.5,  $100 \text{ mW/cm}^2$ ). Segments:  $A > 10$ ,  $J_{0(A>10)} = 6.0 \cdot 10^{-8} \text{ A/cm}^2$ ;  $A = 5$ ,  $J_{0(A=5)} = 1.5 \cdot 10^{-10} \text{ A/cm}^2$ ;  $A = 4$ ,  $J_{0(A=4)} = 7 \cdot 10^{-12} \text{ A/cm}^2$ ;  $A = 3$ ,  $J_{0(A=3)} = 8 \cdot 10^{-18} \text{ A/cm}^2$ .

The  $\eta(\propto V_\eta) - C(\propto J_g)$  dependence character repeats completely the character of the  $J-V_\phi$  dependence (Fig. 9, 10). Both dependencies have alike number of the segments. It is clear from Fig. 9 and 10 that the  $\eta - C$  characteristics without accounting for the resistance  $R_S$  have, practically, the same shape as the dark  $V-J_\phi$  characteristics. Allowing for the series resistance  $R_S$  in a MJ SC bends “down” the  $\eta - C$  characteristic forming a maximum. Presence of the maximum on the  $\eta(\propto V_\eta) - C(\propto J_g)$  characteristic allows determining the series ohmic resistance simulating the whole connecting part of a MJ SC (Fig. 5):  $R_S \approx E/J_{gmax}$ , where  $E \equiv A\varepsilon$ .

## 5 CONCLUSION

As a result of the suppositions and approximations made and investigations of InGaP/GaAs/Ge MJ SCs carried out, the following has been shown:

- segmental character of two main characteristics of a MJ SC: “dark forward current density – voltage”,  $J-V_\phi$ , and “efficiency – sunlight concentration ratio”,  $\eta(\propto V_\eta) - C(\propto J_g)$ ;
- correspondence and similarity of the  $J-V_\phi$  and  $\eta(\propto V_\eta) - C(\propto J_g)$  characteristics of the MJ SCs;
- rated “virtual” characteristic of a InGaP/GaAs/Ge SC plotted with the use of the characteristics of singlejunction Ge, GaAs and GaInP SCs fits well to the experimental  $J-V_\phi$  characteristic of a monolithic GaInP/GaAs/Ge SC;
- reduction of the equivalent electrical circuit of a MJ SC to the equivalent electrical circuit of a “virtual” singlejunction SC.

An analytical correlation between the introduced definition “efficiency” voltage  $V_\eta$  and the generation current  $J_g$  or between the efficiency  $\eta$  and the sunlight concentration ratio  $C$  has been obtained. Correspondence between the  $J-V_\phi$  and the  $\eta(\propto V_\eta) - C(\propto J_g)$  characteristics has been demonstrated experimentally. Both these characteristics have a shape of a continuous function comprised of segments with smoothed-out bend between neighbouring segments until there is no effect of the series resistance ( $R_S$ ). The  $J-V_\phi$  characteristics of the monolithic InGaP/GaAs/Ge SCs have been obtained and the parameters  $J_0$  and  $A$  of the segments used for calculating and plotting the  $\eta - C$  characteristics have been determined. The rated and the experimental  $\eta - C$  characteristics of the monolithic InGaP/GaAs/Ge SC coincide well (Fig. 9, 10), that indicates the validity of the suppositions and approximations made. It should be noted that a good coincidence has been obtained for InGaP/GaAs/Ge MJ SCs, where complete photocurrent  $J_g$  matching between PV Ge, GaAs and InGaP  $p-n$  junctions not always takes place.

In the experimental  $J-V$  characteristic of MJ SCs, a sharp current rise has been observed and analyzed, which is caused by charge carrier injection from PV GaAs and InGaP  $p-n$  junctions into a region of the tunneling junction connecting them.

The analytical correlation between the efficiency and the sunlight concentration ratio  $\eta - C$ , is valid also for other singlejunction SCs and photocells, in particular

based on Si, Ge and other A<sup>III</sup>B<sup>V</sup>.

It has been shown experimentally (fig. 6, 9, 10) that, at the low concentrated ( $C < 10$ ) sunlight, the recombination mechanism of current flow gives the main contribution into the current (dark and photogenerated) in GaAs and InGaP  $p-n$  junctions. At high ( $C > 100$ ) sunlight concentration ratios, that is given by the diffusion mechanism in PV Ge, GaAs and InGaP  $p-n$  junctions. A slightly illumination ( $C < 1$ ), the tunneling-trapping mechanism in the considered Ge, GaAs and InGaP PV  $p-n$  junctions forming a MJ SC has an effect.

Revealing and analysis of the segments and other peculiarities in the experimental  $J-V_\phi$  and  $\eta(\propto V_\eta) - C(\propto J_g)$  dependencies of MJ SCs is an available and useful method for testing and determining the parameters affecting the MJ SC efficiency.

## 6 ACKNOWLEDGMENTS

The authors express their gratitude to Il'menkov G.V., Kozlova M.S., Kalyuzhny N.A., Mintairov S.A. for useful remarks and help in the work.

This work was supported by the Russian Foundation for Basic Research, grants no. 09-08-00412 and no. 09-08-12202.

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