ON DEPENDENCE OF THE MULTIJUNCTION InGaP/GaAs/Ge, InGaP/GaAs SOLAR CELL EFFICIENCY ON THE SUNLIGHT CONCENTRATION.

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ABSTRACT: Correlation between two main characteristics of multijunction (MJ) solar cells (SC) and their segment character have been studied theoretically and experimentally: the dark voltage – current characteristic and the characteristic corresponding to it: efficiency – concentration ratio, \( \eta \propto -C(\alpha J) \). The characteristics have been investigated in two steps. At the first step, an idealization has been done, which did not take into account of tunnel \( p^+ - n^- \) junctions, series resistance of the structure, and other elements. The idealized characteristics have the shape of an uninterrupted function comprised of specific segments. For each segment, an analytical formula expressing the generated current through the efficiency has been derived. The formula derivation has been done at the condition of generated current matching (equality of generated currents in photovoltaic junctions of a MJSC). The analytical \( \eta \propto -C(\alpha J) \) characteristic segment is set by the same two parameters (preexponential factor \( J_0 \) and diode coefficient \( A \)) as of the corresponding dark \( V - J \) characteristic segment. Therefore, the performance connectorless \( \eta \propto -C(\alpha J) \) characteristic is unambiguously associated with the initial connectorless dark \( V - J \) characteristic. At the second step, the effect of the MJSC connecting part has been taken into account, which was simulated by selected ohmic resistance. The connectorless \( \eta \propto -C(\alpha J) \) characteristic have been calculated and plotted segment by segment. A good fit between the rated and experimental dependencies have been obtained for dual-junction InGaP/GaAs SCs. A acceptable fit between the calculated and experimental data has been obtained for triple-junction InGaP/GaAs/Ge SCs, in spite of there is no generated current matching.

Keywords: Solar Cell Efficiencies, Multijunction Solar Cell, Tandem, Concentrator Cells, Gallium Arsenide Based cells.

1 INTRODUCTION

The dependence efficiency (\( \eta \)) of solar cells (SCs) on the sunlight concentration ratio (\( C \)) has a maximum [1,2,3].

It has been shown [3] that, on the portion preceding the maximum, quite important are: first, the recombination mechanism (Sah, Noyce, Shockley), second pure-tunneling mechanism and thermo-tunneling (excess by the Esaki terminology [4,5] defect-dependent mechanism. The effect of the excess (tunneling) mechanism rises in decreasing the illuminance, and, in some cases, it becomes rather noticeable at non-concentrated (\( C = 1 \)) and weakened (\( C < 1 \)) solar illumination.

The subject of the present work is the dark \( J - V \) characteristics of the MJSC photovoltaic (generator) part (Fig.1). The photovoltaic (generator) part means an equivalent circuit containing only photovoltaic \( p-n \) junctions connected in series. Correspondingly, the connecting part of the MJSC is an equivalent circuit containing tunnel junctions, series resistance, etc (Fig.1).

The main characteristic practically important for solar cells is the sunlight concentration ratio vs efficiency (\( C - \eta \)). It is derived just from the initial \( (J - V)_0 \) characteristic [3]. Correspondingly, the \( C - \eta \) characteristic shape depends on that the initial \( (J - V)_0 \) characteristic, which is determined by the current passage mechanism in the SC photovoltaic \( p-n \) junctions.

The multijunction structures for triple-junction InGaP/Ga(In)As/Ge and dual-junction InGaP/Ga(In)As solar cells have been fabricated by the MOCVD technique.

2 MAIN CHARACTERISTICS OF A MULTIJUNCTION SOLAR CELL.

2.1 Dark and light connectorless current-voltage characteristics of a separate photovoltaic \( p-n \) junction.

2.1.1. Segment approximation of the dark connectorless current-voltage characteristic. The dark forward current-voltage characteristic \( (J - V)_0 \) of each photovoltaic (PV) \( p-n \) junction comprising a MJSC can be describes by a sum of three current components [6]: diffusion one (the diode coefficient \( A_d = 1 \)), recombination one (\( A_r = 2 \)) and tunneling (defect-dependent) one (\( A_t > 2 \)).
The voltage boundary between segments is:

\[ V_i = \frac{AKT}{q} \ln \left( \frac{J_i}{J_{i-1}} - 1 \right) \]  

(2)

At such a segment approximation, the whole \((J - V)_e\) characteristic is an uninterrupted function comprised of separate segments. At \( V_e < \frac{AKT}{q} \) or \( J >> J_0 \) one may neglect unity in the brackets of (1) and (2) expressions. Then, the voltage boundary between segments is:

\[ V_i = 2 \frac{KT}{q} \ln \frac{J_i}{J_{i-1}} \]  

(3a)

where \( i, k \) are indices of neighboring segments. The current boundary:

\[ J_k = J_{i-1} \left( 1 + \frac{A_{ij}}{A_{ik}} \right) \]  

(3b)

2.1.2 Light current-voltage characteristic segment.

The light characteristic segment is obtained from a corresponding dark segment (2):

\[ J_e = J_e - J \]  

(5)

where \( J_e \) is the generation current density, which proportional to the illumination intensity measured, in particular, by the sunlight concentration ratio \( C \). \( J \) is the density of the external current, \( J_e \) is the density of the forward current (in this case internal one) of a \( p-n \) junction. In the expression (5), the direction of \( J \) and \( J_e \) are chosen in such a way that they are positive on the load portion of the light characteristic \((J - V)\). Substitution of (5) in (2) (but before \( J_e \) substituted for \( J \)) gives the light voltage-current characteristic segment:

\[ V_e = \frac{AKT}{q} \ln \left( \frac{J_e - J}{J_0} + 1 \right) \]  

(6)

2.2 Dark and light current-voltage characteristic of the MJSC photovoltaic part.

2.2.1 Light connectorless voltage-current characteristic segment.

The light voltage-current characteristic segment of the photovoltaic (generation) part of a MJSC is obtained by summing voltage of the segments (6) \( PV \) \( p-n \) junctions:

\[ V_e = \sum_{i=1}^{n} V_i = \frac{kT}{q} \ln \left( \prod_{i=1}^{n} \left( \frac{J_i - J}{J_0} + 1 \right)^{A_{ii}} \right) \]  

(7)

where \( i \) is the number of a \( PV \) \( p-n \) junction, \( n \) is the amount of \( PV \) \( p-n \) junctions. At the condition of equality of generation current densities \( (J_i - J) >> J_0 \), and else, neglecting unity in the brackets of the expression (7) (which is valid at \( V_e > \frac{AKT}{q} \) or \( (J_i - J) >> J_0 \), we obtain \((J - V)_e\) characteristic of the segment:

\[ V_e = \frac{AKT}{q} \ln \left( \frac{J_e - J}{J_0} \right) \]  

(8a)

or

\[ J = J_e - J_0 \exp \frac{qV_e}{AKT} \]  

(8b)

where \( A = A_1 + A_2 + \ldots + A_n \)

(8c)

\[ J_0 = \sqrt[\sum \{A_i\}]{J_0^A J_0^B \ldots J_0^N} \]  

(8d)

Note that the segment (8a) has exactly the same shape as the segment of a separate \( PV \) \( p-n \) junction (6), but has another (resulting) value of the diode coefficient and the preexponential factor.

Figure 2: Segmental shape of dark current-voltage characteristic of photovoltaic \( p-n \) junctions being a part of a multijunction SC. Segmental shape of dark current-voltage characteristic of dual-junction and triple-junction SCs consisting of the \( p-n \) junctions.

In particular, for the recombination-diffusion boundary:

\[ J_{rd} = \frac{J_e}{J_0}; \quad V_{rd} = 2 \frac{KT}{q} \ln \frac{J_e}{J_0} \]  

(4)

Smoothing off angles between the segments is performed, if required, by summing (1) the current components.
2.2.2. Dark connectorless current-voltage characteristic.

The dark characteristic segment is obtained from (8a) or (8b) at a zero generation current, \( J_g = 0 \), and, according to (5), \( J_0 = -J \)

\[
V_\phi = \frac{AKT}{q} \ln \left( \frac{J_e}{J_0} + 1 \right) \quad (9a)
\]

or \( J_e = J_0 \left( \exp \left( \frac{qV_e}{AKT} \right) - 1 \right) \quad (9b) \)

Unity in brackets is added to impair a standard form to the expression (9a) and (9b). Note that the segment (9a) has the same shape as the segment of separate PV p-n junction (2). As a result, the whole dark current-voltage characteristic of the MJSC photovoltaic part has the same shape as the whole dark characteristic of a separate PV p-n junction: uninterrupted function comprised of segments. Smoothing out the angles between segments can be performed as it is done in (1) by means of summing current of the segments (9b).

However, it is necessary for this that the diode coefficients of the segments would decrease with current. But this, as distinct from a separate PV p-n junction, is not always valid for the photovoltaic part of a MJSC.

2.2.3. Dependence of the open circuit voltage on the generated current (proportional to the sunlight concentration ratio), \( V_{oc} = J_g \varepsilon \). At the open circuit, \( J_g = 0 \), \( V_{oc} = V_{oc} \). Then it follows from (8b) for a segment:

\[
J_g = J_0 \exp \left( \frac{qV_{oc}}{AKT} \right) \quad (10)
\]

The segment (10) of the \( J_g - V_{oc} \) characteristic coincides with the segment (9b) of the dark \( J - V_{oc} \) characteristic, i.e. it has alike preexponential factor \( J_0 \) and diode coefficient \( A \) values. Correspondingly, the whole \( J_g - V_{oc} \) characteristic coincides with the dark \( J - V_{oc} \) one, and also had a shape of uninterrupted function comprised of segments (p. 2.2.2).

2.3 Correlation between the optimum efficiency and the generated current (proportional to the sunlight concentration ratio), \( \eta = J_g \varepsilon \).

At conventional approach [1,2], the operating voltage \( V_n \) is determined from the condition of the maximum power on the external load. To perform this, the derivative of this power is taken equal to zero, \( \frac{d}{dV_e} (J_v J_e) = 0 \), where the segment current \( J \) is presented by the formula (8b). Then to determine \( V_{\text{max}} = V_n \), it is necessary to find out the transcendent equation [7] root:

\[
V_n = V_{oc} + E \ln \left( 1 + \frac{V_n}{E} \right) \quad (11a)
\]

or

\[
\left( 1 + \frac{V_n}{E} \right) \exp \left( \frac{V_n}{E} \right) = \exp \left( \frac{V_{oc}}{E} \right) \quad (11b)
\]

where \( E = \frac{AKT}{q} \). In correspondence with (8b)

\[
J_g = J_0 - J_0 \exp \left( \frac{V_{oc}}{E} \right) \quad (12)
\]

or, substituting \( J_0 \) from (10) into (12) and then \( \exp \left( \frac{V_{oc}}{E} \right) \)

from (11b),

\[
J_g = J_0 \left( \frac{V_{oc}}{V_{oc} + E} \right) \quad (13)
\]

The optimum efficiency is:

\[
\eta = \frac{J_g V_n}{P_{oc}} = \frac{J_g}{P_{oc}} \left( \frac{V_n^2}{V_{oc} + E} \right) = \frac{V_n}{V_{con}} \quad (14)
\]

where \( P_{oc} \) is the power density of radiation incident on the receiving surface of a SC. At a fixed spectrum (AM0, AM1.5, laser and other), the ratio \( \frac{P_{oc}}{J_g} \) does not depend on the illumination intensity. For this reason, the entire dependence of the efficiency on the generated current is confined in the efficiency voltage \([8,9]\)]

\[
V_\eta = \frac{V_n}{V_{oc} + E} \quad (15)
\]

By means of this quadratic (with respect to \( V_n \)) equation, the operating voltage \( V_n \) is expressed through the efficiency voltage \( V_{\eta} \varepsilon \eta \):

\[
V_n = \frac{1}{2} \left( V_{\eta} \varepsilon \eta + \sqrt{V_{\eta}^2 (V_{\eta} + 4E)} \right) \quad (16)
\]

The final required analytical correlation between the efficiency voltage \( V_{\eta} \varepsilon \eta \) and the generated current density \( J_{g} \varepsilon \eta \) is derived from the combination of (10), (11b) and (16):
The entire $J_{sc}(x_C) - V_{oc}(x_\eta)$ characteristic is comprised of segments of the (17) shape and represents uninterrupted function with breaks. To smooth out the angles, summing the generation current of the segments (17) is used. However, as in the case the dark $I-V_p$ characteristic (p.2.2.2), this is not always possible to do.

2.4 Approximate accounting for the effect of the connector part on efficiency.

2.4.1. Modified (with accounting for the connector) analytical correlation between the efficiency and generated current (proportional to the sunlight concentration ratio), $\eta - J_{sc}(x_C)$.

The voltage on the external load, $V_s$, is less than the voltage generated by photovoltaic $p-n$ junction, $V_{sc}$, by the value equal to the voltage on the SC connecting part, $V_s = V_{sc} - V_s$. One may neglect this if $V_s < E = \frac{AKT}{q}$.

Such a condition is fulfilled at low sunlight concentrations ($C < 10$) in the structures under investigation.

For approximate accounting for the “excess” voltage $V_s$, the following is supposed. First, the entire connecting part (including the tunneling $p^-n^+$ junctions) is simulated by one series ohmic resistance $R_c$. Second, it is supposed that accounting for $R_c$ does not, practically, change the operating current $J_{sc}$ which, at $V_{sc} >> E$, is approximately equal to $J_{oc}$ as follows from (13). Only the operating voltage $V_{sc}$ decreases and, according to (15), the efficiency voltage $V_s$ by the value about $J_{sc}R_c$. Such an assumption is based on that the shape of the connector (not allowing for the effect of the connector) current-voltage characteristic is close to a rectangular one (Fig. 3). It is clear from the figure that three characteristic points (rectangle vertex $v$, operating point $m$ and efficiency point $\eta$) are close to each other and remain to be close in accounting for resistance. It is also seen that the voltage $V_s$ decreases by the $I_{sc}R_c$ value. Hence, the efficiency voltage $V_{sc}$ (proportional to the efficiency $\eta = \frac{V_{sc}}{V_{conv}}$) decreases by approximately the same value.

So, the approximate accounting for the effect of the connector on the efficiency lies in a drop of the connectorless efficiency voltage from the formula (12) by the $J_{sc}R_c$. In an analytical form, the modified correlation $\eta = \eta(V_s) - C(x_{J_{sc}})$ is following one.

$$J_{sc} = J_{sc} \left[1 + \frac{V_{oc} + \sqrt{V_{oc}^2 + 4E}}{2E} \right] \exp \left[ \frac{V_{oc} + \sqrt{V_{oc}^2 + 4E}}{2E} \right]$$

(18a,b) between $J_{sc}$ and $V_{oc}$ attains an implicit form.

Beside, a maximum of $J_{ge} \approx \frac{E}{R_c}$ appeared.

2.4.2. On a maximum of the $\eta - J_{sc}(x_C)$ characteristic.

In the maximum point $\frac{d\eta}{dJ_{sc}} = 0$, hence, (formula (14))

$$\frac{dV_{sc}}{dJ_{sc}} = \frac{dV_{oc}}{dJ_{sc}} - R_c;$$

(19)

The derivative $\frac{dV_{oc}}{dJ_{sc}} = \left(\frac{dJ_{sc}}{V_{sc}}\right)^{-1}$ is found from the equation (18b), being simplified at $V_s >> 4E$ (than $V_{sc} >> V_s + E$):

$$J_{sc} \approx e \cdot J_{oc} \cdot \frac{V_{oc}}{E} \cdot \exp \left(\frac{V_{oc}}{E}\right)$$

(20)

where $e \approx 2.72$ is the basis of natural logarithms. As follows from (20), at using inequality $V_{sc} >> 4E$, $\frac{dJ_{sc}}{dV_{sc}} \approx \frac{J_{sc}}{E}$.

Therefore, it is seen from (19) that the generation current $J_{sc}$ in the efficiency maximum point $J_{oc} \approx \frac{E}{R_c}$.

3 DARK CURRENT-VOLTAGE CHARACTERISTICS (EXPERIMENT).

3.1 Dark forward current-voltage characteristic of Ge, GaAs and InGaP $p-n$ junctions.

These characteristic have been obtained in the forward current density range of $10^{-11} \text{A/cm}^2$ at room temperature. Three components of the forward current were obtained in GaAs and InGaP homo $p-n$ junctions: diffusion one ($A=1$), recombination one ($A=2$) and tunneling one ($A=2$) (Fig. 4).
The tunneling type components revealed at the current densities lower than \(10^{-6} \text{ A/cm}^2\) (GaAs), \(10^{-8} \text{ A/cm}^2\) (InGaP), \(10^{-10} \text{ A/cm}^2\) (Ge).

Values of the diffusion and recombination preexponential factors are presented in Table 1.

Table 1:

<table>
<thead>
<tr>
<th>homo (p-n) junctions</th>
<th>preexponential factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ge</td>
<td>recombination, (J_r)</td>
</tr>
<tr>
<td></td>
<td>((1-2) 10^{10}) cm²</td>
</tr>
<tr>
<td>GaAs</td>
<td>((2-5) 10^{11}) cm²</td>
</tr>
<tr>
<td>InGaP</td>
<td>((3-7) 10^{14}) cm²</td>
</tr>
</tbody>
</table>

The preexponential factors obtained by approximation of the dark forward current-voltage characteristic by three exponents are \(J_{(A=4)} = (1-9) 10^{13} \text{ A/cm}^2, J_{(A=3)} = (0.5-9) 10^{23} \text{ A/cm}^2\).

3.2 Dark forward current-voltage characteristic of the InGaP/GaAs tandems.

These characteristic have been obtained in the forward current-voltage range of \((10^{-11}-10^{-5}) \text{ A/cm}^2\) at room temperature.

The forward current-voltage characteristic of the dual-junction monolithic InGaP/GaAs tandem consists of 3 portion and is approximated by a sum of three components with the diode coefficients \(A = 2, A = 4\) and \(A > 4\) (Fig. 5). The upper portion \((A = 2)\), as follows from p.2.2.1. and 2.2.2, is formed of the diffusion portion of the GaAs and InGaP \(p-n\) junctions, i.e. has a pure diffusion origine. The middle portion \((A = 4)\) is formed of corresponding recombination portions and has a pure recombination origin. On the lower portion \((A = 4)\), the tunneling components has a significant contribution, at least of one homo \(p-n\) junction. The portion of mixed origine \((A = 3)\) (the diffusion portion of one homo \(p-n\) junction and the recombination portion of another homo \(p-n\) junction) did not appear. This means that it short compared with the smoothing interval between the upper \((A = 2)\) and middle \((A = 4)\) portions. Besides, this indicates additionally the proximity of the current recombination-diffusion boundaries \(J_{rd}\) for InGaP and GaAs homo \(p-n\) junction (Table 1).

Figure 5: Characteristics of dual-junction InGaP/GaAs SC: a - experimental dark current-voltage characteristic laced with the corresponding \(V_{OC}-J_{SC}\), where the preexponential factor \(J_{(A=2)} = 1.6 \times 10^{15} \text{ A/cm}^2\) on the segment with \(A = 4\) and the preexponential factor \(J_{(A=2)} = 3.1 \times 10^{-23} \text{ A/cm}^2\) on the segment with \(A = 2\); b - experimental and rated dependences of the efficiency on generated current \(\eta - J_g\): 1 - connectorless characteristic, 2 - rated \(\eta - J_g\) characteristic by formulas (18), 2 - experimental \(\eta - J_g\) dependence (circles), \(J_{(C=1)} = 13.5 \text{ mA/cm}^2, \text{AM0}\).

Figure 6: Characteristics of triple-junction InGaP/GaAs/Ge SC: a - experimental dark current-voltage characteristic laced with the corresponding \(V_{OC}-J_{SC}\), where the preexponential factor \(J_{(A=3)} = 6 \times 10^{11} \text{ A/cm}^2\) on the segment with \(A = 5\). This segment is formed of GaAs and InGaP wideband \(p-n\) junction recombination segments and Ge \(p-n\) junction diffusion segment; b - experimental and rated dependences of the efficiency on generated current \(\eta - J_g\): 1 - connectorless characteristic, 2 - rated \(\eta - J_g\) characteristic by formulas (18), 3 - experimental \(\eta - J_g\) dependence (circles), \(J_{(C=1)} = 13.7 \text{ mA/cm}^2, \text{AM1.5}\).
3.3 Dark forward current-voltage characteristic of triple-junction InGaP/GaAs/Ge structures.

Measurements were carried out at room temperature in the current density range of \((10^3 - 10^{11})\, A/cm^2\). The dark characteristic consists of two portions and are approximated by a sum of two exponents with diode coefficients \(A=5\) and \(A=5\) (Fig.6). This figure shows the dark \(J-V\) characteristic laced with the generation current-open circuit voltage characteristic \(J_{\text{g}} - V_{\text{sc}}\), \(J_{\text{g}} = J_{\text{g}}\).

The upper portion \((d=5)\), as follows from p.2.2.1. and 2.2.2., is formed of the diffusion portion of the Ge \(p-n\) junction, the recombination portion of the GaAs \(p-n\) junction and the recombination portion of the GaInP \(p-n\) junction, i.e. it has a mixed origin. On the lower portion \((A=5)\), the tunneling components contribution is significant, at least in one generating \(p-n\) junction.

The preexponential factor on the mixed portion \(J_{\text{g(dif)}} = (5 - 7)\times 10^{11}\, A/cm^2\). This value agrees with the formula (8c), in which the \(J_{\text{g}}(\text{Ge})\), \(J_{\text{g}}(\text{GaAs})\), \(J_{\text{g}}(\text{InGaP})\) values are taken from Table 1.

4 CHARACTER OF THE DEPENDENCE OF THE EFFICIENCY ON THE GENERATING CURRENT AND SUNLIGHT CONCENTRATION RATIO, \(\eta - J_{\text{g}}(\alpha C)\).

4.1 Calculation of the \(J_{\text{g}}(\alpha C) - V_{\text{g}}(\alpha \eta)\) characteristic.

The experimental dark current-voltage characteristic gives a feasibility to calculate (by the formula (18) at the condition of generation current matching of photovoltaic \(p-n\) function comprising the PV part of a MJSC) and plot the generated current-efficiency voltage characteristic, \(J_{\text{g}} - V_{\text{sc}}\), for each segment. This characteristic is proportional to the sunlight concentration ratio – efficiency voltage characteristic, \(C(\alpha J_{\text{g}}) - \eta(\alpha V_{\text{g}})\).

The proportionality coefficient between \(J_{\text{g}}\) and \(C\) is the generation current at one sun, \(J_{\text{g}}=V_{\text{sc}}\), which is determined experimentally, since it confines at \(C=1\) with the short circuit current, \(J_{\text{sc}}\approx J_{\text{c}}\). It is necessary for such a coincidence that the condition \(J_{\text{g}}=V_{\text{sc}}\) would be fulfilled, which is obeyed at \(C=1\).

The proportionality coefficient between \(V_{\text{g}}\) and \(\eta\) is the conversion voltage, \(V_{\text{g}}=\frac{P_{\text{max}}}{J_{\text{g}}}(3.2.6)\) which does not depend out the illumination intensity (at a fixed spectrum) and is usually determined at one sun \((C=1)\), \(V_{\text{g}}=\frac{P_{\text{max}}}{J_{\text{c}}}=136.6\, mW/cm^2\) (AM0) or 100 mW/cm^2 (AM1.5d).

To obtain the entire \(\eta(\alpha V_{\text{g}}) - C(\alpha J_{\text{g}})\) characteristic from segments, their generation currents were summed: \(J_{\text{g}} = \sum J_{\text{g}_i}\), where \(i\) is the segment index. Such a summation results is smoothing off the angels between segments. It is admissible, if the diode coefficients of the dark \((J - V)\) characteric decrease with increasing of the current, which is obeyed (Fig. 5b).

4.2 Dependence efficiency – sunlight concentration ratio, \(\eta(\alpha V_{\text{g}}) - C(\alpha J_{\text{g}})\), for the InGaP/GaAs tandem.

The \(C\) dependence calculated by the formula (18) and plotted fits well to the experimental one at \(R_{\text{c}} \approx 0.18\, Om\cdot cm^2\) (Fig. 5b). The coincidence indicates the validity of the following assumption and approximation done in obtained the equation (18):

1. The photovoltaic \(p-n\) junctions are generation current matched: \(J_{\text{g}} = J_{\text{g}_1} = \ldots = J_{\text{g}_\text{d}} = J_{\text{g}}\).
2. The whole connecting part of a MJSC (connector) was approximated by the series ohmic resistance, \(R_{\text{c}}\).
3. The efficiency voltage (proportional to the efficiency) \(V_{\text{g}}(\alpha \eta)\) calculated in the connectorless approximation \((R_{\text{c}} = 0)\) was decreased by the \(J_{\text{g}}R_{\text{c}}\) value to account for the negative effect of the connector on the efficiency.

The character of the \(\eta(\alpha V_{\text{g}}) - C(\alpha J_{\text{g}})\) dependence (Fig. 5b) repeats that the dark current-voltage dependence, (Fig. 5a). Both dependencies have similar numbers of segments. Allowing for the series resistance bends the \(\eta - J_{\text{g}}\) characteristic “down” (towards smaller \(\eta\)) forming a maximum.

Presence of the maximum on the \(\eta(\alpha V_{\text{g}}) = C(\alpha J_{\text{g}})\) characteristic may be used for estimating the series resistance approximating the connecting part of a MJSC (connector) \(R_{\text{c}} \approx \frac{E}{J_{\text{g}}},\) where \(E = \frac{AKT}{q}\).

4.3 Efficiency – sunlight concentration ratio dependence for a triple-junction InGaP/Ga(In)As/Ge structure.

The \(\eta - J_{\text{g}}(\alpha C)\) dependence calculated by the formula (18) fits well to the experimental one at \(R_{\text{c}} = 0.085\, Om\cdot cm^2\) and \(J_{\text{g}}(C = 1) \approx 13.7\, mA/cm^2\). The segmental characteristic of the dark \((J - V)\) characteristic agrees with the \(J_{\text{g}}(\alpha C) - V_{\text{g}}(\alpha \eta)\) characteristic character (Fig. 6).

The coincidence indicates the validity of the assumption and approximations done in deriving the (18) equation.

The results of the analytical calculations show that, for triple-junction solar cells, one may use the model with the following three elements of an equivalent current.

1. Selected single generated current close to the generated current of a dual-junction tandem.
2. Selected series ohmic resistance approximating the entire connecting part of a multijunction solar cell.
3. Virtual (generating) \(p-n\) junction; the dark and “inner” (at illumination), current is summed of several exponents with diode coefficients \(A=3, 4, 5\) and \(>5\) (Fig. 1).

5 CONCLUSION.

In the multijunction InGaP/GaAs, InGaP/GaAs/Ge solar cells, analyzed are:

- First, correspondence between two main characteristic: dark current – voltage, \(J - V\), and efficiency (proportional to the efficiency voltage) – sunlight concentration ratio (proportional to the generated current), \(\eta(\alpha V_{\text{g}}) - C(\alpha J_{\text{g}})\).
• Second, segmental character of these two characteristics.

The following assumption and approximations have been used:
1) segment approximation (dividing to segments) of these two characteristics,
2) generated current matching between PV p-n junctions comprising the PV part of a solar cell (which, strictly speaking, is not valid for InGaP/GaAs/Ge solar cells),
3) approximation of the connecting part of a SC (including tunneling p-n junctions) by a ohmic resistance, \( R_s \),
4) correction (decrease) of the efficiency voltage (proportional to the efficiency) by the \( J_g R_s \) value to account for the negative effect of \( R_s \) on the efficiency.

At these assumptions and approximations, an analytical correlations between the efficiency voltage, \( V_\eta \), and the generated current, \( J_g \), and, hence, between the efficiency, \( \eta \), and the sunlight concentration ratio, \( C \), has been derived. There exists a correlation between \( J-V \) and characteristics. At small dark and photogenerated currents, until the effect of the simulation series ohmic resistance falls upon \( (JR_s \frac{J}{R_s} \propto C) < \varepsilon (\frac{J}{R_s}) \) characteristics. At small dark and photogenerated currents, until the effect of the simulation series ohmic resistance falls upon \( (JR_s \frac{J}{R_s} \propto C) < \varepsilon (\frac{J}{R_s}) \) characteristics. At small dark and photogenerated currents, until the effect of the simulation series ohmic resistance falls upon \( (JR_s \frac{J}{R_s} \propto C) < \varepsilon (\frac{J}{R_s}) \) characteristics. At small dark and photogenerated currents, until the effect of the simulation series ohmic resistance falls upon \( (JR_s \frac{J}{R_s} \propto C) < \varepsilon (\frac{J}{R_s}) \) characteristics.

The connectorless \( J-V \) characteristics of the dual-junction InGaP/GaAs solar cells have been obtained, and the segment parameters \( (J_0, A) \) have been determined, which were used for calculating and plotting the \( \eta - J_g (\propto C) \) characteristic. The rated \( \eta - J_g (\propto C) \) characteristic fits to the experimental ones (Fig.5) which indicates that the four assumptions and approximations done in calculating the \( \eta - J_g (\propto C) \) characteristic are satisfactory. Moreover, such a coincidence is present for the triple-junction InGaP/GaAs/Ge solar cells, in spite of there is no generated current matching between the Ge PV p-n junction and the GaAs and InGaP PV p-n junctions.

The obtained analytical correlation between the efficiency and the generated current (proportional to the sunlight concentration ratio) is also valid for single-junction SCs (in particular, Si SCs) and other photo cells (in particular, Ge thermophotovoltaic cells), and even to larger extent. Actually, for the single-junction cell, first, there is no need for current matching and, second, the connecting part is represented by a real, but not simulating, series ohmic resistance \( R_s \).

One of the practically useful results of the work is the following one. It has been established that, at the nonconcentrated \( (C=1) \) and weakly concentrated \( (C=10) \) sunlight in the case of GaAs and GaInP p-n junctions, the recombination component gives the main contribution to the dark and inner (at illumination) currents. At high \( (C=100) \) ratios in the case of Ge, GaAs, GaInP p-n junctions, the main contribution is given by the diffusion component. At weakened \( (C=1) \) solar illumination, the effect of the tunneling (excess) mechanism in all three PV p-n junctions often takes place.